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ScienceDirect
Engineering Structures 29 (2007) 3134–3148
ENGINEERING STRUCTURES
www.elsevier.com/locate/engstruct

3D homogenized limit analysis of masonry buildings under horizontal loads

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Received 25 May 2006; received in revised form 19 December 2006; accepted 2 March 2007
Available online 8 April 2007

Abstract

The current confidence levels in the ability to provide buildings with adequate resistance to horizontal actions do not easily apply to historic and existing masonry structures. Limit analysis is often not sufficient for a full structural analysis under seismic loads, but it can be profitably used in order to obtain a simple and fast estimation of collapse loads. Often, the limit analysis of ancient masonry structures is used in the context of several simplifications, the assumptions about the collapse mechanisms being the most relevant. Aiming at a more general framework, a micro-mechanical model developed previously by the authors for the limit analysis of isolated in- and out-of-plane loaded masonry walls is extended here and utilized in the presence of coupled membrane and flexural effects. In the model, the elementary cell is subdivided along its thickness in several layers, where fully equilibrated stress fields adopting a polynomial expansion are assumed. The continuity of the stress vector on the interfaces between adjacent sub-domains and anti-periodicity conditions on the boundary surface are further imposed. Linearized homogenized surfaces for masonry in six dimensions are obtained and implemented in a FE limit analysis code, and two 3D case studies are analyzed making use of the kinematic theorem of limit analysis. From the results, the approach proposed is validated and its usefulness for solving engineering problems is demonstrated.

Keywords: Masonry; Limit analysis; Kinematic approach; 3D analysis

1. Introduction

It has been shown that the high vulnerability of historical masonry buildings to horizontal actions is mostly due to the absence of adequate connections between the various parts, especially when wooden beams are present both in the floors and in the roof [1]. This characteristic leads to overturning collapses of the perimeter walls under seismic horizontal acceleration and combined in- and out-of-plane failures. The evaluation of the ultimate load bearing capacity of masonry buildings subjected to horizontal loads is a fundamental task in their design and safety assessment. Simplified limit analysis methods are usually adopted by practitioners for safety analyses and design of strengthening [2]. However, codes of practice, such as for instance the recent Italian O.P.C.M. 3431 [3, 4], require a static nonlinear analysis for existing masonry buildings, in which a limited ductile behavior of the elements is taken into account, featuring failure mechanisms such as rocking, shear and diagonal cracking of the walls. Nowadays, several models for the analysis of masonry buildings are at our disposal, but the approach based on the use of averaged constitutive equations seems to be the only one suitable for employment in a large scale finite element analysis [5]. Heterogeneous approaches based on a distinct representation of bricks and joints seem to be limited to the study of panels of small dimensions, due to the large number of variables involved in a nonlinear finite element analysis. Therefore, alternative strategies based on macro-modeling have been recently developed in order to tackle engineering problems (see Lourenço et al. [6]). Obviously, macro-approaches require a preliminary mechanical characterization of the model, which has to be derived from experimental data from laboratory or *in situ* testing [7].

In this framework, homogenization techniques can be used for the analysis of large scale structures. Such techniques take into account at a cell level the mechanical properties of

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0141-0296/$ - see front matter
doi:10.1016/j.engstruct.2007.03.003

constituent materials and the geometry of the elementary cell, allowing the analysis of entire buildings through standard finite element codes. Furthermore, the application of homogenization theory to the rigid-plastic case [8] requires only a reduced number of material parameters and provides significant information at failure, such as limit multipliers, collapse mechanisms, and at least on critical sections, the stress distribution [9].

In this paper, the micro-mechanical model presented by the authors in [9,10] and [11] for the limit analysis of in- and out-of-plane loaded masonry walls respectively, is extended and utilized in the presence of coupled membrane and flexural effects. In the model, the elementary cell is subdivided along its thickness into several layers. For each layer, fully equilibrated stress fields are assumed, adopting polynomial expressions for the stress tensor components in a finite number of sub-domains. The continuity of the stress vector on the interfaces between adjacent sub-domains and suitable anti-periodicity conditions on the boundary surface are further imposed. In this way, linearized homogenized surfaces in six dimensions (polytopes) for in- and out-of-plane loaded masonry are obtained. Such surfaces are then implemented in a FE limit analysis code for the analysis at collapse of entire 3D structures, and meaningful examples of technical relevance are discussed in detail.

In Section 2, the micro-mechanical model adopted for obtaining masonry homogenized polytopes is recalled, whereas in Section 3 the FE upper bound approach is presented. The method is based on a triangular discretization of the structure, so that the velocity field interpolation is linear inside each element. Plastic dissipation can occur for in-plane actions both in the continuum and in the interfaces. On the other hand, since the velocities interpolation is assumed linear inside each element, the curvature rate tensor is equal to zero for each triangle, and out-of-plane dissipation can take place only at the interfaces between adjoining triangles.

Two meaningful structural examples are treated in detail in Section 4. The first numerical simulation refers to the prediction of the ultimate seismic load of a two story masonry building of dimensions 7.32 × 7.32 × 7.14 m (length × width × height). The building was experimentally tested by Yi et al. [12] under cyclic loads in the inelastic range at Georgia Tech, USA. The second example consists of an ancient house already studied by De Benedictis et al. in [13] within an extensive survey project coordinated by Giuffrè [2] of the entire Ortigia (Italy) city center.

The reliability of the proposed model is assessed through previously presented numerical results [14], and through comparisons with results obtained by means of standard nonlinear FE approaches.

2. In- and out-of-plane homogenized failure surfaces

A masonry wall Ω constituted by a periodic arrangement of bricks and mortar disposed in running bond texture is considered, as shown in Fig. 1a. As pointed out by Suquet in [8] from a general point of view, homogenization techniques combined with limit analysis can be applied for the evaluation of the homogenized in- and out-of-plane strength domains S^{hom} of the masonry. Under the assumptions of perfect plasticity and its associated flow rule for the constituent materials, and in the framework of the lower bound limit analysis theorem, S^{hom} can be derived by means of the following (nonlinear) optimization problem (see also Fig. 1):

$$S^{\text{hom}} = \left\{ \max(\mathbf{M},\mathbf{N}) \middle| \begin{cases} \mathbf{N} = \dfrac{l}{|Y|}\displaystyle\int_{Y\times h} \boldsymbol{\sigma}\,\mathrm{d}V & \text{(a)} \\ \mathbf{M} = \dfrac{l}{|Y|}\displaystyle\int_{Y\times h} y_3\boldsymbol{\sigma}\,\mathrm{d}V & \text{(b)} \\ \operatorname{div}\boldsymbol{\sigma} = \mathbf{0} & \text{(c)} \\ [[\boldsymbol{\sigma}]]\mathbf{n}^{\text{int}} = \mathbf{0} & \text{(d)} \\ \boldsymbol{\sigma}\mathbf{n} \text{ anti-periodic on } \partial Y_l & \text{(e)} \\ \boldsymbol{\sigma}(\mathbf{y}) \in S^m\ \forall \mathbf{y} \in Y^m;\ \boldsymbol{\sigma}(\mathbf{y}) \in S^b\ \forall \mathbf{y} \in Y^b & \text{(f)} \end{cases} \right\} \quad (1)$$

where:

- $\mathbf{N}$ and $\mathbf{M}$ are the macroscopic in-plane (membrane forces) and out-of-plane (bending moments and torsion) tensors;
- $\boldsymbol{\sigma}$ denotes the microscopic stress tensor;
- $\mathbf{n}$ is the outward versor of ∂Y_l surface, Fig. 1a;
- ∂Y_l is defined in Fig. 1a;
- $[[\boldsymbol{\sigma}]]$ is the jump of micro-stresses across any discontinuity surface of normal $\mathbf{n}^{\text{int}}$, Fig. 1c;
- S^m and S^b denote respectively the strength domains of mortar and bricks;
- Y is the cross-section of the 3D elementary cell with $y_3 = 0$ (see Fig. 1) $|Y|$ is its area, V is the elementary cell volume, h represents the wall thickness, and $\mathbf{y} = \begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix}$ are the assumed material axes;
- condition (1(c)) imposes a micro-equilibrium with zero body forces, usually neglected in the framework of the homogenization theory;
- anti-periodicity condition (1(e)) requires that the stress vectors $\boldsymbol{\sigma}\mathbf{n}$ are opposite on opposite sides of ∂Y_l, Fig. 1c, i.e. $\sigma^{(m)}\mathbf{n}_1 = -\sigma^{(n)}\mathbf{n}_2$;
- Y^m and Y^b represent mortar joints and bricks respectively, see Fig. 1.

In order to solve Eq. (1) numerically, the simple admissible and equilibrated micro-mechanical model proposed in [10] is adopted. The unit cell is subdivided into a fixed number of layers along its thickness, as shown in Fig. 1b. For each layer, out-of-plane components σ_{i3} $(i = 1, 2, 3)$ of the micro-stress tensor $\boldsymbol{\sigma}$ are set to zero, so that only in-plane components σ_{ij} $(i, j = 1, 2)$ are considered active. Furthermore, σ_{ij} $(i, j = 1, 2)$ are kept constant along the Δ_L thickness of each layer, i.e. in each layer $\sigma_{ij} = \sigma_{ij}(y_1, y_2)$. For each layer, one-fourth of the representative volume element is sub-divided into nine geometrical elementary entities (sub-domains), so that the entire elementary cell is sub-divided into 36 sub-domains (see [10] for further details and Fig. 1b).

For each sub-domain (k) and layer (L), polynomial distributions of degree (m) in the variables (y_1, y_2) are a priori assumed for the stress components. Since the stresses are polynomial expressions, the generic ijth component can be written as follows:

$$\sigma_{ij}^{(k,L)} = \mathbf{X}(\mathbf{y})\mathbf{S}_{ij}^{(k,L)T} \quad \mathbf{y} \in Y^{(k,L)} \quad (2)$$