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Stress concentration due to shear lag in continuous box girders

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Abstract

Quite a few researches on shear lag effect in box girder have been reported in the past, and many of them employed the finite element method. The past researchers, however, do not seem to have paid much attention to the influence of the finite element mesh on the shear lag, although the shear lag effect in terms of stress concentration can be quite sensitive to the mesh employed in the finite element analysis. In addition, most of the researchers on the shear lag have focused on simply supported girders and cantilever girders, while continuous girders have been dealt with in very few researches. The present study investigates the shear lag effect in a continuous box girder by using the three-dimensional finite element method. The whole girder is modeled by shell elements, and an extensive parametric study with respect to the geometry of a box girder is carried out. The influence of finite element mesh on the shear lag is carefully treated by the multimesh extrapolation method. Based on the numerical results thus obtained, empirical formulas are proposed to compute stress concentration factors that include the shear lag effect.

Keywords: Shear lag; Continuous box girders; Stress concentration; Three-dimensional finite element analysis

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1. Introduction

Although normal stress in the longitudinal direction produced by bending deformation is assumed to be uniform across flange width in the elementary beam theory, it is not so in reality if the flange width is large. This phenomenon, known as the shear lag, has been studied for many years. A concise but excellent literature review of research on the shear lag is available in Tenchev [1]. Even in recent years, the subject has attracted many researchers and quite a few papers have been published [2–6].

Although much research has been done on the problem in the past, a discrepancy in numerical results is observed in the literature, an illustration of which is given by Lertsima et al. [6] for the case of simply supported girders. The discrepancy seems to be attributable to the factors that have considerable influence on the shear lag but have been overlooked.

Lertsima et al. studied the shear lag of a simply supported box girder by the three-dimensional finite element analysis, using shell elements [6]. Loads were applied in multiple ways. Much attention was paid to finite element meshes as well: in short, the multimesh extrapolation method [7] was utilized so as to reduce discretization error and thus enhance accuracy of the results due to the finite element analysis. An extensive parametric study was then conducted and empirical formulas were proposed.

Continuous girders are quite common structures in practice. Engineers dealing with ordinary box girders for highway bridges and buildings need not be daunted by the stress concentration due to shear lag. However, there are indeed some special cases of short stocky members, so that design codes provide formulas to account for the shear lag effect [8, 9]. Nevertheless, there appear to be very few research results available in the literature on the shear lag effect of continuous girders. Besides, Japan [8] and Eurocode 3 [9] yield very different shear lag effects, which will be shown later in this paper.

Against the background of the above information, the three-dimensional finite element analysis of a continuous box girder by shell elements is carried out to investigate stress concentration due to the shear lag in the present study. With the

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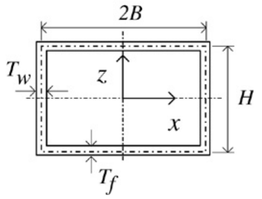
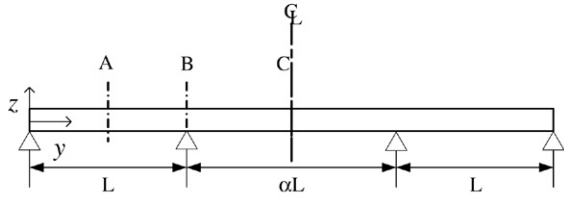
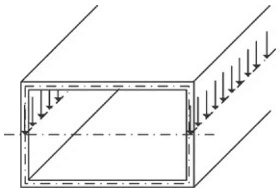


(a) Cross section.



(b) Side view.



(c) Distributed load.

Fig. 1. Three-span continuous box girder model.

multimesh extrapolation method [7], the analysis is performed to produce reliable numerical results. An extensive parametric study is conducted and empirical formulas are proposed to deal with the shear lag phenomenon in continuous box girders. In all the analyses, a well-known finite element program, MARC [10], is used.

2. Continuous box girder model

Three-span continuous box girders under uniformly distributed load are analyzed. The symbols employed in the present study for describing the structural geometry are illustrated in Fig. 1. For the design of a continuous girder, the stress distributions in the cross sections under large bending moment are important. Therefore, in the present study we focus on three cross sections of Sections A–C shown in Fig. 1(b): Section A is under the largest bending moment in the exterior span, Section B is at the interior support (under the largest negative bending moment) and Section C is at the center of the girder, which is under the largest bending moment in the interior span. Following Lertsima et al. [6], a uniformly distributed load is applied as a line load along the centerline of the web as shown in Fig. 1(c). Due to symmetry, only a quarter of the girder (a half of the cross section and a half of the girder length) need be analyzed. Therefore, the symmetric conditions, i.e., no displacement in y-axis and no rotations about x- and z-axes, are imposed on Section C, while only the displacement in z-axis is suppressed at the end of the girder and Section B. The material property is assumed to be isotropic linear elastic with Young's modulus 206 GPa and Poisson's ratio 0.3.

Table 1
Comparison of K_c values ($H/L = 0.2$; $B/H = 2.0$; $\alpha = 1.0$; $T_f/T_w = 1.0$)

Literature	K_c		
	Section A	Section B	Section C
Japan [8]	2.83	4.64	3.51
Eurocode 3 [9]	2.18	4.74	2.66

The stress concentration factors in Sections A–C can be evaluated by the formula given in the design codes [8,9]. For a box girder with $B/H = 2.0$, $H/L = 0.2$, $T_f/T_w = 1.0$ and $\alpha = 1.0$, those values are computed and presented in Table 1, where K_c stands for the stress concentration factor defined by the ratio of the maximum normal stress in the flange to that of the elementary beam theory. Significant discrepancy is recognized, suggesting the necessity of the further study of the shear lag in a continuous girder.

In the present study, the continuous box girders are analyzed by the three-dimensional finite element method, using 4-node shell elements. In particular, Element 75 (Bilinear Thick Shell Element) is used, and the nodal stress is evaluated as an average of the stresses in the elements sharing the node [10]. Although the finite element method is very versatile and powerful, caution must be used since the results may depend largely on the finite element mesh employed in the analysis, which is especially so when stress concentration is dealt with. The dependency on the finite element mesh is attributable to the discretization error. In Lertsima et al. [6], this issue was looked into numerically, and the multimesh extrapolation method was employed to reduce the discretization error in the evaluation of the stress concentration due to shear lag. It is noted that the stress concentration factors thus obtained are very close to those obtained by the adaptive finite element method [6]. The multimesh extrapolation method is also used herein, and the numerical procedure is briefly explained in what follows:

Fig. 2(a) shows the normal-stress distributions in the upper flange at the mid-span of a simply-supported box girder ($B/H = 1.0$, $H/L = 0.2$, $T_f/T_w = 1.0$) that was dealt with in Lertsima et al. [6]. In this figure, σ is the normal stress obtained by the three-dimensional finite element analysis, while σ_{beam} is the normal stress due to the elementary beam theory. Needless to say, σ_{beam} is constant across the flange width. Using the 4-node shell elements, four finite element meshes of Meshes A–D are employed herein. All the elements in each mesh are rectangular and every element in the box girder is made a quarter in the process of refining the mesh from Meshes A–D: an illustration of Meshes A–D is presented in Fig. 3. The total numbers of elements are 1920, 7680, 30,720 and 122,880 for Meshes A–D, respectively. Fig. 2(a) illustrates not only the shear lag phenomenon but also the dependence of the stress distribution on the finite element mesh. As expected, the dependence is stronger at the edge of the flange where the largest stress concentration takes place. At the same time, the tendency of stress convergence is observed, as the size of the finite element becomes smaller.

The stresses at four points in the upper flange obtained by the present finite element analysis are presented in Fig. 2(b).