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Engineering Structures 28 (2006) 1997–2009

ENGINEERING STRUCTURES

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Lateral buckling analysis of thin-walled laminated composite beams with monosymmetric sections

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Received 18 October 2005; received in revised form 14 March 2006; accepted 15 March 2006
Available online 6 May 2006

Abstract

Lateral buckling of thin-walled composite beams with monosymmetric sections is studied. A general geometrically nonlinear model for thin-walled laminated composites with arbitrary open cross-section and general laminate stacking sequences is given by using systematic variational formulation based on the classical lamination theory. All the stress resultants concerning bar and shell forces are defined, and nonlinear strain tensor is derived. General nonlinear governing equations are given, and the lateral buckling equations are derived by linearizing the nonlinear governing equations. Based on the analytical model, a displacement-based one-dimensional finite element model is developed to formulate the problem. Numerical examples are obtained for thin-walled composite beams with monosymmetric cross-sections and angle-ply laminates. The effects of fiber orientation, location of applied load, modulus ratio, and height-to-span ratio on the lateral buckling load are investigated. The torsion parameter and a newly-defined composite monosymmetry parameter are also investigated for various cases.

Keywords: Thin-walled composites; Lateral buckling; Nonlinear theory; Classical lamination theory

1. Introduction

Fiber-reinforced composite materials have been used over the past few decades in a variety of structures. Composites have many desirable characteristics, such as high ratio of stiffness and strength to weight, corrosion resistance and magnetic transparency. Thin-walled structural shapes made up of composite materials, which are usually produced by pultrusion, are being increasingly used in many engineering fields. In particular, the use of pultruded composites in civil engineering structures awaits increased attention.

Thin-walled composite structures are often very thin and have complicated material anisotropy. Thus, warping and other secondary coupling effects should be considered in the analysis of thin-walled composite structures. The theory of thin-walled open section members made of isotropic materials was first developed by Vlasov [1] and Gjelsvik [2]. For fiber-reinforced composites, Bauld and Tzeng [3] presented a nonlinear model for thin-walled composites by extending Gjelsvic's [2] formulation to the balanced symmetric laminated composite materials. However, the formulation is somewhat not consistent in the sense that it uses coordinate mapping when developing nonlinear stresses instead of variational formulation. Bhaskar and Librescu [4] presented a nonlinear theory of composite thin-walled beams accounting for finite displacements and arbitrary large twist angles. Recently, Fraternali and Feo [5] developed a small strain and moderate rotation theory of laminated composite thin-walled beams by generalizing the classical Vlasov theory. They presented a beam model accounting for axial, bending, torsion and warping deformations. Omidvar and Ghorbanpoor [6] developed a nonlinear finite element model for thin-walled open-section structural members made of laminated composites with symmetric stacking sequence.

As thin-walled composites are being increasingly used as structural members, the stability becomes a major consideration due to their slenderness. The lateral stability of thin-walled open section members made of isotropic materials have been studied by many researchers [7,8]. The lateral buckling analysis of beams subjected to various loads were studied by Bleich [8]. Kitipornchai et al. [9] presented a buckling analysis for a

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doi:10.1016/j.engstruct.2006.03.024

monosymmetric beam under moment gradient. Trahair [10] carried out extensive investigations of the lateral buckling of beams.

The lateral buckling of composite beams have been studied by several researchers [11–16]. Kabir and Sherbourne [11] studied lateral-torsional buckling of I-section composite beams using the Rayleigh–Ritz method. They also studied the shear strain effects in lateral stability of thin-walled composite beams [12]. Lin et al. [13] studied the buckling problem of thin-walled composite structural members by the finite element method. Davalos and Qiao [14] presented a combined analytical and experimental evaluation of the flexural-torsional and lateral-distortional buckling of fiber-reinforced plastic composite with wide-flange beams. Roberts [15] investigated the influence of shear deformation on buckling of pultruded composites. Recently, Lee et al. [16] proposed an analytical model for predicting lateral buckling moments for composite I-section beams.

Most of the work concerning the lateral stability of thin-walled composite beams are focused on I-sections. Kabir and Sherbourne [17] proposed an analytical solution for predicting the lateral buckling capacity of composite channel-section beams. More recently, Lee et al. [18] presented an analytical model which treats the lateral buckling of channel-section composite beams. The model was capable of predicting accurate buckling loads and modes for various configurations. As far as the authors are aware, the lateral buckling analysis of a composite beam with monosymmetric section bent about the asymmetric axis has not yet been treated in the open literature.

In the present paper, the analytical model developed by the authors [16,18] is extended to a geometrically nonlinear model for thin-walled laminated composites with arbitrary open cross-section and general laminate stacking sequences. A systematic approach based on the variational formulation is used to formulate the theory. All the stress resultants concerning bar and shell forces are defined, and nonlinear strain tensor is given. General nonlinear governing equations are given, and finally, lateral buckling and prebuckling equations for arbitrary cross-section are derived by linearizing the nonlinear governing equations. A one-dimensional finite element model is developed to formulate the lateral buckling of a thin-walled composite beam with monosymmetric section. Numerical examples are obtained for thin-walled composite beams of monosymmetric sections with angle-ply laminates. The effects of fiber orientation, location of applied load, modulus ratio, and span-to-height ratio on the buckling behavior of the composite beams are studied. The torsion parameter and a newly-defined composite monosymmetry parameter are also investigated for various cases.

2. Displacement fields

The theoretical developments presented in this paper require two sets of coordinate systems which are mutually interrelated. The first coordinate system is the orthogonal Cartesian coordinate system (x, y, z), for which the x and y axes lie in the plane of the cross section and the z axis parallel to the longitudinal axis of the beam. The second coordinate system is the local plate coordinate (n, s, z) as shown in Fig. 1, wherein the n axis is normal to the middle surface of a plate element, while the s axis is tangent to the middle surface and is directed along the contour line of the cross section. The (n, s, z) and (x, y, z) coordinate systems are related through an angle of orientation θ as defined in Fig. 1. Point P is called the pole axis, through which the axis parallel to the z axis is called the pole axis.

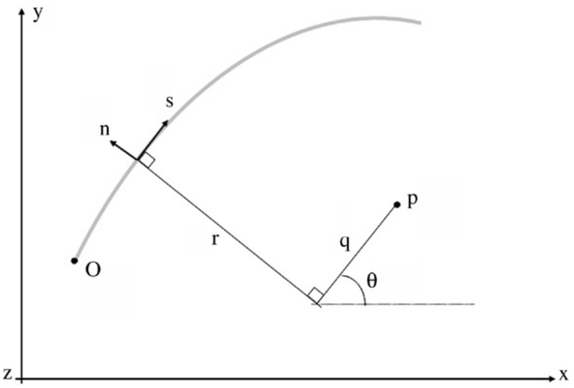

Fig. 1. Definition of coordinates in thin-walled open sections.

To derive the analytical model for a thin-walled composite beam, the following assumptions are made:

1. The contour of the thin wall does not deform in its own plane. Although many thin-walled sections fail due to local buckling, the local buckling effect is neglected in this study.
2. The linear shear strain $\bar{\gamma}_{sz}$ of the middle surface is zero in each element.
3. The Kirchhoff–Love assumption in classical plate theory remains valid for laminated composite thin-walled beams.

According to assumption 1, the midsurface displacement components $\bar{u}, \bar{v}$ at a point A in the contour coordinate system can be expressed in terms of a displacement U, V of the pole P in the x, y directions, respectively, and the rotation angle Φ about the pole axis,

$$\bar{u}(s, z) = U(z)\sin\theta(s) - V(z)\cos\theta(s) - \Phi(z)q(s) \quad (1a)$$

$$\bar{v}(s, z) = U(z)\cos\theta(s) + V(z)\sin\theta(s) + \Phi(z)r(s). \quad (1b)$$

These equations apply to the whole contour. The out-of-plane shell displacement $\bar{w}$ can now be found from assumption 2 (also known as the Vlasov assumption). For each element of middle surface

$$\bar{\gamma}_{sz} = \frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial s} = 0. \quad (2)$$

After substituting for $\bar{v}$ from Eq. (1a) and considering the following geometric relations,

$$\mathrm{d}x = \mathrm{d}s\cos\theta \quad (3a)$$

$$\mathrm{d}y = \mathrm{d}s\sin\theta. \quad (3b)$$

Eq. (2) can be integrated with respect to s from the origin to an arbitrary point on the contour

$$\bar{w}(s, z) = W(z) - U'(z)x(s) - V'(z)y(s) - \Phi'(z)\omega(s) \quad (4)$$