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Flexural behaviour of externally prestressed beams. Part I: Analytical model

Chee Khoon Ng[a,*], Kiang Hwee Tan[b]

[a] *Faculty of Engineering, Universiti Malaysia Sarawak, 94300 Kota Samarahan, Sarawak, Malaysia*
[b] *Department of Civil Engineering, National University of Singapore, Blk E1A, #07-03, 1 Engineering Drive 2, Singapore 117576, Singapore*

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Abstract

Two features that distinguish an externally prestressed beam from an otherwise internally bonded, prestressed beam are: (i) the tendon stress depends on the overall deformation of the beam; and (ii) the tendons are free to move relative to the section depth, resulting in eccentricity variations defined as second-order effects. This paper presents a simple "pseudo-section analysis" method which accounts for second-order effects in simply supported, externally prestressed beams subjected to two symmetrically applied concentrated loads. The proposed method predicts the load–deflection curve and provides explicit expressions for the tendon stress, which can be used to evaluate the moment capacity of the beam by section analysis based on the bond reduction coefficient in strain compatibility.

Keywords: Analysis; Beam; External prestressing; Flexure; Strength; Second-order effects

1. Introduction

External prestressing refers to a post-tensioning method in which the tendons are placed on the outside of a structural element to facilitate flexural resistance. It may be efficiently utilized in the construction of segmental box-girder bridges as well as in the strengthening of existing concrete beams [1,2]. However, there has been relatively little documentation on the analysis and design of externally prestressed structures [3].

One of the reasons which leads to the complexity in the analysis of externally prestressed beams can be attributed to the eccentricity variations of external tendons under load, commonly referred to as second-order effects. That is, under the application of external loads, a concrete beam deforms with a nonlinear profile while the external tendons remain rectilinear in between anchorages and/or deviators, as shown in Fig. 1(a). This results in a relative movement of the external tendons with respect to the centroid of the concrete section in between the anchorages and/or deviators, as a result of which the flexural capacity of the beam is reduced [4–9].

Fig. 1(b) shows two schematic load–deflection curves of an externally prestressed beam. The solid line represents the load–deflection curve of the beam where second-order effects are neglected. If second-order effects are taken into account, the stiffness of the beam is reduced and the ultimate strength is relatively lower as shown by the dashed line. Mutsuyoshi et al. [7] tested a series of externally prestressed beams with a span-to-depth ratio of about 21 and reported that the reduction in beam strength due to second-order effects can be as high as 16%. In another theoretical study by Alkhairi and Naaman [5], the eccentricity variation was reported to be more significant in beams with span-to-depth ratios greater than 24 and strength reduction as high as 25% can be observed for beams with a span-to-depth ratio of 45.

Several investigators [5–8,10] have attempted to consider the variation of eccentricity in their models for the analysis of externally prestressed beams. Typically, the effective tendon eccentricity at any location was related to the deflections of the adjacent deviators. By dividing the beams into finite sections and considering compatibility of member deformation and equilibrium of forces and moments, the total elongation of the external tendons for an applied load can be taken as the integral of the concrete strains at the level of the tendon. Of the available models, Alkhairi and Naaman's model [5] distinguishes itself from the others in that it considers an additional moment induced by shear. On the other hand, Mutsuyoshi et al. [7] have developed prediction equations based

* Corresponding author. Tel.: +60 82 670525; fax: +60 82 672317.
E-mail address: ckng@feng.unimas.my (C.K. Ng).

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Notation

A_{ps}	area of prestressing or external tendons
A_s	area of internal tension reinforcement
A'_s	area of internal compression reinforcement
A_{tr}	transformed area of beam cross section
b	beam width
b_w	beam web width
c	neutral axis depth
d_{ps}	tendon depth
d_{ps0}	initial tendon depth
$d_{ps,u}$	tendon depth at ultimate limit state
d_s	effective depth of internal tension reinforcement
d'_s	effective depth of internal compression reinforcement
e	tendon eccentricity
$e_{0(x)}$	function defining the eccentricity along the span
e_m	initial tendon eccentricity at midspan
e_s	tendon eccentricity at support
e_u	tendon eccentricity at ultimate limit state
E_c	modulus of elasticity of concrete
E_{ps}	modulus of elasticity of prestressing or external tendons
f'_c	concrete cylinder compressive strength
f_{cu}	concrete cube compressive strength
f_{pe}	effective prestress of prestressing or external tendons
f_{ps}	ultimate tendon stress of prestressing or external tendons
f_{py}	yield strength of prestressing or external tendons
f_r	modulus of rupture of concrete
f_y	yield strength of internal tension reinforcement
f'_y	yield strength of internal compression reinforcement
h	overall beam height
h_f	flange thickness of beam
I	moment of inertia of the beam cross section
I_{cr}	moment of inertia for cracked cross section taken about the neutral axis
I_e	effective moment of inertia
I_{tr}	moment of inertia for transformed cross section taken about the neutral axis
k_s	constant for consideration of second-order effects
L	effective beam span
L_d	distance from beam support to deviation point
L_q	distance between two symmetrically applied concentrated loads
L_s	distance from beam support to loading point
m	bending-moment equation due to a unit load at the section under consideration
m_{dev}	moment due to one unit load at the deviation point
m_{mid}	moment due to one unit load at midspan
M	applied moment or moment within the constant moment region
M_{cr}	cracking moment
$(M_{cr})_e$	cracking moment due to initial effective prestress
M_{dec}	decompression moment
M_{ecl}	moment corresponding to elastic cracked limit
M_g	moment due to self-weight
M_{ps}	bending-moment equation due to the prestressing force along the span
$M_{(x)}$	bending-moment equation along the span
M_u	ultimate moment of resistance
M_y	moment corresponding to yield load
S_d	distance between two symmetrically placed deviators
Z_b	elastic modulus of the critical section based on the extreme concrete fibre subjected to tensile stress
β_1	compression stress block depth factor
$\delta_{dev}^{(-)}$	camber at deviation point due to prestressing force
$\delta_{dev}^{(+)}$	deflection at deviation point due to applied load
δ_{mid}	midspan deflection
$\delta_{mid}^{(-)}$	midspan camber due to the prestressing force
$\delta_{mid}^{(+)}$	midspan deflection due to applied load
$\varDelta$	relative upward displacement
ΔM_{cr}	cracking moment due to stress increase in external tendons
$\Delta\varepsilon_{ps}$	strain increase in external tendons
ε_c	strain in the top concrete fibre
ε_{ce}	pre-compression strain in the prestressing or external tendons
ε_{cu}	strain in the top concrete fibre at ultimate limit state
ϕ_u	curvature at ultimate limit state
$\varOmega$	bond reduction coefficient for the linear elastic uncracked regime
$\varOmega_c$	bond reduction coefficient for the cracked regime
$\varOmega_u$	bond reduction coefficient at ultimate limit state

on numerical experiments which may be used to evaluate the tendon stress and effective tendon eccentricity at the ultimate flexural strength limit state.

The “member analysis” approach is rather tedious and it may be simplified by a “pseudo-section analysis”, the development of which is the subject of this paper. The approach proposed herein accounts for second-order effects and predicts the complete response of a simply supported beam subjected to two symmetrical concentrated loads, with up to two deviators placed symmetrically about the midspan of the beam.

The proposed analytical method in this paper is developed for monolithic beams only. However, in an experimental study carried out by Aparicio et al. [11] with five monolithic and three segmental beams tested in either flexure or combined flexure and shear, it was found that the behaviour of a segmental closed-joint beam is similar to a monolithic beam. Aravinthan et al. [12] had also investigated monolithic and segmental beams with highly eccentric external tendons. It was also found that the flexural behaviour of a monolithic beam is similar