Fire Safety Journal 82 (2016) 30–36

Contents lists available at ScienceDirect

Fire Safety Journal

journal homepage: www.elsevier.com/locate/firesaf

An analytical approach for estimating uncertainty in measured temperatures of concrete slab during fire

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ARTICLE INFO

Article history:
Received 19 June 2015
Received in revised form
28 January 2016
Accepted 26 March 2016
Available online 1 April 2016

Keywords:
Uncertainty
Heat transfer model
Monte Carlo method
Analytical model
Concrete slab
Fire
Composite floor system

ABSTRACT

To evaluate the fire resistance of concrete slabs used in composite floor systems, the temporal and spatial variation of measured temperatures must be accurately determined. Temperature profiles in a concrete section are a function of concrete thermophysical properties and boundary conditions. However, there can be considerable uncertainty in the estimates typically used for thermophysical properties and boundary conditions in fire. In this study, an analytical approach was developed to compute uncertainties in concrete slab temperatures during exposure to a fire and the results obtained with this approach were verified against those obtained with the Monte Carlo method. A simple 1-D heat flow model was constructed to demonstrate the usefulness of this approach. It is shown that uncertainty in gas temperatures has a substantial effect on the overall uncertainty in computed member temperatures. Also, uncertainties associated with computed concrete temperatures increase with increase in fire exposure time.

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1. Introduction

In order to evaluate the fire resistance of structural members, the temporal and spatial variation of their temperatures during fire exposure must be estimated with reasonable accuracy. The ability to predict time-varying temperature profiles in structural members with high-confidence is necessary for a robust performance-based approach to the fire resistance design of structures. While there have been a number of fire tests conducted to study the thermal behavior of steel components, there is considerably less information available in the literature on the time varying temperature profile in concrete slabs during a fire. The focus of this study is to develop a simple approach for estimating uncertainty in the predicted thermal response of concrete slabs during a fire event.

Temperature profiles in a concrete section during fire exposure depend upon the temperature-dependent thermophysical properties of concrete, and the convective and radiative heat transfer parameters. There can be considerable uncertainty in estimates typically used for these parameters. For example, the thermal conductivity and heat capacity of concrete can vary considerably with the moisture level in concrete [1]. Since the moisture level in concrete changes during the progression of a fire, the thermal conductivity and heat capacity of concrete can vary with time, influencing the overall thermal response of the concrete slab. A sensitivity study can be conducted to determine which of these parameters (thermophysical and heat transfer) most significantly influence the thermal response of the slab. Influential parameters can then be used to quantify the uncertainty in the predicted temperatures.

Uncertainties can be broadly classified into two basic types: aleatoric (random) and epistemic (systematic). Aleatoric uncertainties are due to inherent randomness and cannot be removed by further analysis or testing. For example, fuel load density (MJ/m^2) can be classified as inherently random. On the other hand, epistemic (also known as knowledge-based) uncertainties can be reduced by using improved models or algorithms. Computation of epistemic uncertainties can provide a confidence interval for time-varying estimates of structural temperatures during a fire event.

In order to conduct specific design of concrete structures, it is absolutely essential to know the time varying temperatures of the concrete when exposed to a specific fire event. Although design charts are available that provide thermal gradients in concrete members exposed to a standard fire, one needs to estimate the temperatures and thermal gradients for realistic design fires. Computer based finite element heat transfer analyses are done to ascertain these parameters. However, often simplified analytical approaches are preferred methods for structural engineers to estimate the member temperatures. Such methods are routinely used for estimating temperatures in steel members. For example, the "lumped heat capacity method" is widely used for modeling heat transfer in steel members in fire. Such a simple approach does not exist for

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http://dx.doi.org/10.1016/j.firesaf.2016.03.005
0379-7112/Published by Elsevier Ltd.

concrete members. Thus, the focus of this study is to propose such an approach for concrete slab. It is also useful to develop a simple and practical approach for computing uncertainties in structural temperatures when such simplified analytical approaches are used. This paper discusses a simple analytical approach for modeling 1-D heat transfer in concrete slabs and a methodology for computing uncertainties in predicted slab temperatures.

The study of how output of a model can be assigned to different sources of uncertainty in the model input parameters is defined as sensitivity analysis. One common approach for sensitivity analysis is called local sensitivity analysis, which is analytical or numerical derivative based [2]. In mathematical terms, sensitivity of a cost or objective function with respect to a certain parameter is simply equal to the partial derivative of the cost function with respect to that parameter. Local sensitivity means that all derivatives are taken at a single point. For simple cost function this approach is efficient. However, the situation gets complicated when the derivatives become discontinuous. Local sensitivity analysis is often a OAT (one-at-a-time) technique. In this technique, the effect of a single parameter on the cost function is studied when all other parameters are kept constant. The inherent weakness of this method is that only a small fraction of the design space is explored and that the technique does not provide insight into how interactions among model parameters affect the cost function. This was the subject of a study reported in [3]. Another common approach to sensitivity analysis is a global sensitivity analysis, which is often implemented using Monte Carlo method. Here, a global or representative set of samples are used to explore the design space. This method uses an approach similar to the one used for the design of experiments. For each parameter, multiple values that the parameter can assume are generated often using the specified probability distribution for the parameter. The optimization cost function is then evaluated at each sample point.

In the present study, a local sensitivity study is demonstrated by using an analytical approach for computing the partial derivatives. The uncertainties computed using this approach are compared with those computed using a global sensitivity study conducted using a Monte Carlo method available in ANSYS. The proposed approach for computing uncertainties requires obtaining partial derivatives of temperatures with respect to each uncertain input parameters. Implicit in this approach is the assumption that second order effects of the uncertainties are negligible. Once the proposed analytical approach for computing uncertainties is verified against the Monte Carlo method, such simple approaches can be used for predicting uncertainties during a real fire event. The paper discusses this proposed approach for computing uncertainties in temperatures and provides a comparison between uncertainties computed using a simple analytical approach with those obtained from a Monte Carlo method available in ANSYS [4],[1]. Note that the focus of the present work is to develop a simple approach for computing uncertainty and not on the validation of predicted results. Therefore, uncertainty estimates of certain parameters such as fire temperature were assumed for the sake of demonstrating a proof of concept only.

2. Simplified analytical model for computing uncertainty in 1-D heat transfer

Derivation of analytical equations for transient heat transfer in 3-D geometry is complex for temperature-dependent thermal properties and non-homogeneous and time-dependent boundary conditions. The analytical equation is rather simple for 1-D heat transfer in a semi-infinite solid with Dirichlet type of boundary conditions (Note that the Dirichlet or the first-type boundary condition specifies values a solution needs to take on the boundary of the domain). If we assume that the concrete slab is a semi-infinite body in the direction of its depth, then a simple expression can be used to describe the thermal field as shown below. A thick body can be modeled as a semi-infinite solid if we are interested in the variation of the thermal field near one surface and the temperature at the far end does not change over time from the initial values. Note that the proposed approach is developed as a test case and similar analytical models can be developed for other boundary conditions [5].

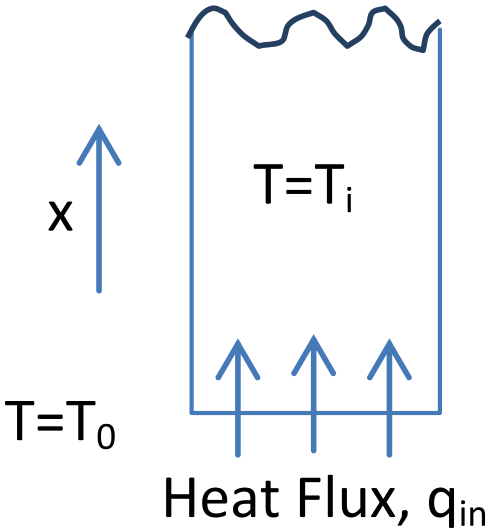

Fig. 1. Schematic representation of 1-D heat flow in a concrete slab (semi-infinite solid) along its depth.

2.1. 1-D heat transfer model

Consider transient 1-D heat flow in a semi-infinite solid (concrete slab) as shown in Fig. 1:

Assume that the concrete slab is heated from one side and the unexposed surface temperature does not change over time. This is a reasonable assumption for the case of a composite floor system exposed to fire underneath the slab. In other words, the concrete slab is assumed to be thermally thick. According to [6], if the temperatures at a given depth within a finite body are not influenced by the heat flow, then the temperatures in the affected portion of the body can be computed using the semi-infinite solid concept. The following inequality must hold for the semi-infinite solution to apply:

$$\frac{h}{\sqrt{4\alpha t}} \geq 0.5 \tag{1}$$

where h is the thickness of the slab, t is the time, and α is the thermal diffusivity. The following expression is valid for 1-D transient heat flow assuming that the thermal properties are independent of temperature (T is the temperature, x is the distance along the heat flow direction e.g., along the slab depth, t is the time):

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2} \tag{2}$$

the boundary and initial conditions are:

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[1] Certain commercial software or materials are identified to describe a procedure or concept adequately. Such identification is not intended to imply recommendation, endorsement, or implication by NIST that the software or materials are necessarily the best available for the purpose.