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Semi-analytical buckling analysis of reinforced concrete columns exposed to fire

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A B S T R A C T

A new semi-analytical procedure is derived for the determination of buckling of the reinforced concrete column exposed to fire. The fire analysis is performed in three separate steps, of which the time development of temperatures in the fire compartment is performed first, followed by the coupled heat and moisture transfer analysis and, finally, by the mechanical analysis. A particular emphasis has been given to the critical buckling time and the remaining critical buckling load at a selected time. For this purpose, a parametric study has been performed by which the influence of different geometric parameters on the buckling load capacity of reinforced concrete columns has been assessed. The results of this study show that the load-carrying capacity of the column reduces significantly with the increasing time of fire exposure and the column slenderness. Moreover, the initial mechanical load has a small, although not negligible effect on the buckling load capacity.

1. Introduction

Limits to a load-carrying capacity of a single slender column are often set by the onset of buckling. As the columns made of reinforced concrete are not as slender as those made of steel and timber, engineers are often prone to assume that the stability check in concrete structures may be omitted. This is not the case, however, if a reinforced concrete structure is exposed to fire. Fire as a structural loading appears to be recently of a particularly strong interest to engineers. During fire the stability of concrete columns is highly endangered. When only the part of the structure is exposed to a fire, compression in the column may largely increase because its thermal expansion deformation is bounded by the surrounding stiff, not thermally exposed elements [1,2]. A sufficiently high load-carrying capacity of a single reinforced concrete column in fire is thus a necessary condition for providing an adequate overall fire resistance of beam-like structures subjected to fire [3]. Since the pioneering work of Euler on buckling of elastic columns [4], the theoretical and experimental research concerning the load-carrying capacity of various kinds of reinforced concrete structures and their responses to the mechanical loads at room temperature has been rich and widespread so far and it still continues [5–8].

At the time being, the theoretical basis for the buckling analysis of purely elastic structures is well established [9–11]. Unfortunately, the linear elastic material models are not sufficiently precise for the mechanical analysis of reinforced concrete structures [8,12]. The first who studied theoretically the buckling stability of elasto-plastic columns appeared to be Engesser [13]. He proved that the material non-linearity can largely reduce the buckling load. Further, he raised the important question how the column unloads, and suggested that the buckling load of an inelastic column must be obtained from Euler's formulae. This question was only correctly resolved later on by Shanley [14] who, with the help of the simple theoretical model and various experiments, showed that the buckling of an elastic-plastic column occurs at the so-called tangent critical load.

It can be observed from literature that the theoretical researches on plastic buckling of reinforced concrete columns are rare. The exact analytical solutions for the buckling loads of a reinforced concrete Euler-type column are presented in, e.g. Krauberger et al. [8], where the effect of the material non-linearity on the buckling load is fully assessed. The authors of [8] found out that the local weakening of columns in a form of cracks in concrete may have a huge impact on the buckling load. The studies concerning the load-carrying capacity of reinforced concrete columns in fire happen to be even less frequent [3,15–18]. These studies employ the two-step mathematical models, where the time distributions of the temperature over the cross-section during fire are determined, in an uncoupled manner, in the first step, and only then the mechanical analysis is made [3]. These models ignore the combined thermo–hygro processes within the concrete sections.

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In determining the load-carrying capacity of the column in fire, the elastic unloading and post-peak softening of plastic material at the elevated temperature is taken into account in these models [19]. The experiments investigating the behaviour of compressed reinforced concrete columns in fire have also been performed for verification, see, e.g. [16,20–22].

The main goal of the present paper is to derive a novel semi-analytical procedure for a simple determination of the buckling load and the critical time of a single reinforced concrete column subjected to an increasing temperature. The paper has, in addition to the introduction, four sections. In the second section, we discuss the concepts of the three-step fire analysis. In the first step, we have to choose the fire scenario. Here, the course of fire could be considered such that a suitable model of a natural fire would be chosen or we could determine one by simply assuming a standard fire curve. The latter gives the variation of the temperature of gases in a surrounding space of the element in time [23]. If more specific gas temperature distributions were required, the use of a suitable CFD commercial computer program would be used. In the next step, which comprises the so-called thermo–hygro analysis, the coupled heat, moisture, vapour and air transport model is observed. In the third step, here called the mechanical analysis, the load-carrying capacity of the reinforced concrete column due to the combined temperature and the mechanical load is obtained. The third section is devoted to numerical examples and parametrical studies. There we present the critical buckling times of columns and the remaining bearing capacity of the column after it has been exposed to a prescribed duration of fire. Here we extensively study the influence of different geometric parameters on the results. The comparison study and the applicability of the present model is presented in fourth section. There we compare the results of numerical models with the experimental ones from the literature [24–26] and also with the graphical readings from the extensive parametric study. The main findings are gathered in the conclusions.

2. Fire analysis of reinforced concrete column

2.1. Preliminaries

We consider a geometrically perfect, initially straight, planar reinforced concrete column of initial length L and a constant cross-section, see Fig. 1. The column is placed in the (X, Z) plane of a spatial right-handed Cartesian coordinate system (X, Y, Z). The undeformed reference axis of the column is assumed to coincide with its centroidal axis. The material coordinate system (x, y, z) coincides initially with the spatial coordinates, and then follows the deformation of the column. Therefore, $x \equiv X$, $y \equiv Y$ and $z \equiv Z$ in the initial, undeformed state. The column is loaded in the X direction with a centric axial compression load P as displayed in Fig. 1, while simultaneously being exposed to a fire of an increasing temperature magnitude, being spatially uniform over all four sides.

Modelling for the fire analysis of a reinforced concrete column consists of three mathematically uncoupled steps. Each of the steps employs its own model, i.e. (i) the fire scenario model, (ii) the heat and mass transfer model and (iii) the mechanical model. In the first step, the time development of gas temperatures in the fire compartment is chosen in accordance with the expected fire severity. Then, in the second step, the coupled heat and mass transport model is observed. Finally, in the third step, the mechanical model is applied to determine the stress–strain field due to the simultaneous effects of mechanical and temperature loads. In what follows only the last two steps are described in detail.

2.2. Heat and mass transfer model

Once the time distribution of a gas temperature in the fire compartment in the first step of the fire analysis has been assumed, a coupled heat and mass transfer (i.e. free water, water vapour and dry air) in heterogeneous concrete can be determined in the second step of the fire analysis.

The heat and mass transfer is described by the system of equations proposed by Davie et al. [27]. Their model considers the transfer of free water, water vapour and dry air caused by pressure and concentration gradients and the conversion of energy. The model accounts for evaporation of water, the liquefaction of water vapour, the dehydration of the chemically bound water, the capillary pressure and adsorbed water.

The set of governing equations thus comprises the mass conservation of the three constituents – free water, water vapour and dry air, and an additional equation requiring the energy conservation to be satisfied:

water conservation:

$$\frac{\partial(\overline{\rho_{FW}})}{\partial t} = -\nabla\cdot\mathbf{J}_{FW} - \dot{E}_{FW} + \frac{\partial(\overline{\rho_D})}{\partial t} \tag{1}$$

water vapour conservation:

$$\frac{\partial(\overline{\rho_V})}{\partial t} = -\nabla\cdot\mathbf{J}_V + \dot{E}_{FW}, \tag{2}$$

air conservation:

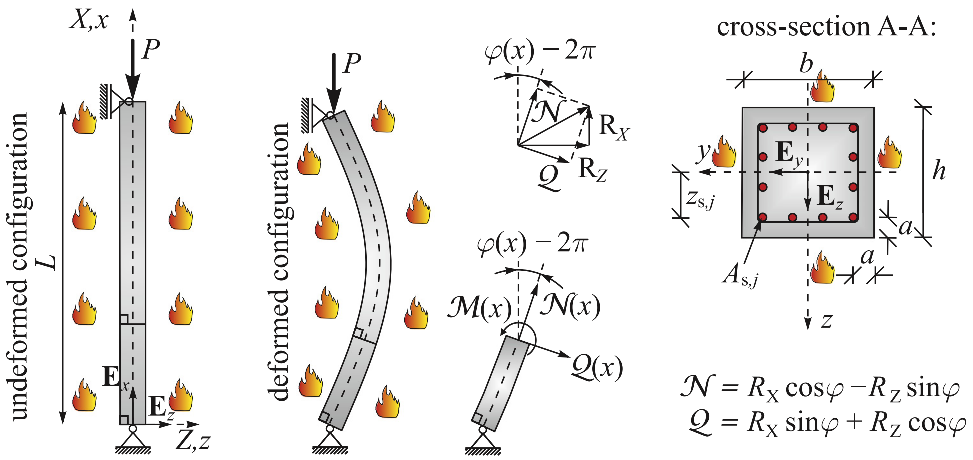

Fig. 1. Geometry and notation for an ideal reinforced concrete column, its undeformed and deformed configuration and a typical cross-section.