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An efficient fiber element approach for the thermo-structural simulation of non-uniformly heated frames

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A B S T R A C T

A computationally efficient fiber element approach has been developed to evaluate the thermo-structural response of non-uniformly heated structural frames. The approach involves the use of two fiber-based elements for conducting the heat transfer and structural simulations. A numerical study is conducted here to evaluate the response of an unprotected beam exposed to a localized fire. Comparisons between the proposed fiber element approach and high-resolution finite element simulations demonstrate that the fiber-based thermal and structural elements collectively offer excellent accuracy. It is further shown that the fiber model substantially improves the computational efficiency by reducing the total number of degrees of freedom in the thermal and structural analyses. The proposed model further simplifies the transfer of thermal data from the heat transfer analysis to the structural model because the thermal and structural elements have the same mesh.

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1. Introduction

Analyses to determine the structural response to fire conditions are typically based on simplified zone models or parametric equations that represent the compartment temperatures during a fire. These temperatures are often applied uniformly on structural elements with the same exposure. While these models are sufficient for many practical applications, there are a number of instances in which it may be desirable to evaluate structural resistance under non-uniform fire exposure. For example, a fire may exhibit localized heating when the fuel is concentrated in a particular region in the compartment or when the fire is in a state of pre-flashover, as is the case during the initial stages of fire development and in relatively large compartments [1,2]. In addition, empirical evidence suggests that natural fires are likely to travel throughout a building rather than burn uniformly, particularly when the building has a large, open floor plan [3]. In these instances, a performance-based assessment of the structure may entail a comprehensive study to evaluate the structural resistance given a more probable type of fire hazard, such as a localized fire or traveling fire scenario.

At present, analysts must rely on high-resolution, three-dimensional (3D) finite element simulations to sequentially evaluate the thermal and structural response of non-uniformly heated structures. The use of such methods in large 3D structures can be numerically unstable and computationally expensive. Alternatively, one can simplify the thermal response of the structure in a manner that is compatible with more efficient frame elements. For example, Franssen et al. [2] demonstrated that the 3D heat transfer equations in beams and columns can effectively be decoupled into a series of 2D heat transfer analyses and the temperature data can subsequently be passed into a structural model utilizing frame elements. While this approach overcomes some of the computational limitations, it is somewhat cumbersome because it requires multiple 2D analyses to be carried out before an accurate prediction of the temperature profile is determined. Furthermore, it does not facilitate the transfer of temperature data from the heat transfer model to the structural model because the two analyses may require different meshes.

To overcome existing limitations, a novel fiber element approach was developed to simulate the thermo-structural response of a frame structure subjected to non-uniform thermal loads. While the use of fiber elements for structural simulation in fire is not new (e.g., refer to earlier works by [4–6]), the problem of transferring high-resolution temperature data from a thermal analysis to a structural model continues to be problematic due to significant mesh disparities. This limitation inhibits advancement towards the efficient analysis of structures exposed to non-uniform heating despite evidence that structural performance may

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Nomenclature

a	fire growth constant
A	cross-sectional area of fiber (i, j)
$[b]$	force interpolation function matrix
$[B]$	first derivative of the shape function matrix with respect to x (i.e., $d[N]/dx$)
C	specific heat
C_0	specific heat of air at ambient (1.0 kJ/kg K)
$[c]$	heat capacity matrix for the heat transfer element
$[c_{i,j}]$	heat capacity matrix for fiber (i, j)
$\{d\}$	array of nodal deformations in the reduced structural element
D	diameter of the fuel package
$\{D\}$	array of nodal displacements in the full structural element
E	tangent modulus for fiber (i, j)
$[f]$	flexibility matrix for the structural element
$[f_s]$	section flexibility matrix for the structural element
F_{1x}, F_{2x}	nodal forces in the structural element's local x-direction
F_{1y}, F_{2y}	nodal forces in the structural element's local y-direction
g	gravitational acceleration (9.81 m/s^2)
h	heat transfer coefficient
h_r	radiation heat transfer coefficient
$[h_{i,j}]$	convection heat transfer matrix for fiber (i, j)
H_B	vertical distance from the base of the fire to the bottom flange
H_C	vertical distance from the base of the fire to the top flange
H_W	vertical distance from the base of the fire to the center of the web
i	integer representing the location of fiber (i, j) with respect to the local y-axis
i^+, i^-	symbols denoting fiber boundaries between grid points in the y-direction (i.e., $i \pm 1/2$)
j	integer representing the location of fiber (i, j) with respect to the local z-axis
j^+, j^-	symbols denoting fiber boundaries between grid points in the z-direction (i.e., $j \pm 1/2$)
k	thermal conductivity
$k_{i^\pm}$, $k_{j^\pm}$	effective conductivities at the fiber interfaces
$[k]$	conductivity matrix for the heat transfer element; reduced stiffness matrix for the structural element
$[k_{i,j}]$	conductivity matrix for fiber (i, j)
$[k_{i^-}]$	AFC matrix for fiber (i,j): contribution of fiber $(i-1, j)$
$[k_{i^+}]$	AFC matrix for fiber (i,j): contribution of fiber $(i+1, j)$
$[k_{j^-}]$	AFC matrix for fiber (i,j): contribution of fiber $(i, j-1)$
$[k_{j^+}]$	AFC matrix for fiber (i,j): contribution of fiber $(i, j+1)$
$[k_s]$	section stiffness matrix for the structural element
L	element length
L_B	flame tip length along the bottom flange
L_C	flame tip length along the top flange
L_W	flame tip length along the web
$M(x)$	internal moment in the structural element
M_1, M_2	nodal moments in the structural element
n	number of fibers
$N(x)$	internal axial force in the structural element
$[N]$	shape function matrix
P	axial force
q''	heat flux
q''_b	prescribed heat flux
Q	internal heat generation per unit volume
$\dot{Q}$	heat release rate
Q^*	dimensionless parameter
r	radial distance from the center of beam
$\{r\}$	array of thermal loads for the heat transfer element; array of nodal forces in the reduced structural element
$\{r_{i,j}\}$	array of thermal loads for fiber (i, j)
$\{r_h\}$	thermal loads due to convective boundary conditions
$\{r_q\}$	thermal loads due to prescribed fluxes
$\{r_Q\}$	thermal loads due to internal heat generation
$\{r_s\}$	array of section forces in the structural element
$\{R\}$	array of nodal forces in the full structural element
S	boundary surface
t	time
T	$T(x, y, z, t)$; temperature
T_0	temperature of air at ambient (293 K)
$T_{i,j}$	$T_{i,j}$ (x, t); temperature of fiber (i, j)
T_{sur}	temperature of the surroundings (for radiation exchange)
T_∞	fluid temperature
$\{T\}$	array of nodal temperatures for the heat transfer element
$\{\dot{T}\}$	first derivative of nodal temperatures with respect to time (i.e., $\partial\{T\}/\partial t$)
$\{T_{i,j}\}$	nodal temperatures for fiber (i, j)
u	axial deformation
u_1, u_2	nodal displacements in the structural element's local x-direction
v	transverse deformation
v_1, v_2	nodal displacements in the structural element's local y-direction
w	non-dimensional distance that is used in the fire model; magnitude of distributed load in the structural model
$y_{i,j}$	y-coordinate of fiber (i, j)
$z_{i,j}$	z-coordinate of fiber (i, j)
z'	virtual source correction
Δy	height of fiber (i, j)
Δz	width of fiber (i, j)
ε	emissivity
$\varepsilon_{i,j}$	strain in fiber (i, j)
ε_{th}	thermal strain in fiber (i, j)
ε_{res}	residual strain in fiber (i, j)
ρ	mass density
ρ_0	density of air at ambient (1.2 kg/m^3)
θ_1, θ_2	nodal rotations in the structural element
σ	Stefan-Boltzmann constant (5.67×10^{-8} W/$m^2\cdot$K^4)

significantly be influenced by spatially varying temperatures [3,7–12]. At the core of the proposed fiber element approach is a new fiber heat transfer element that utilizes a combination of finite element and finite difference methods to solve the 3D conduction heat transfer equations in a robust and highly efficient manner [13]. The heat transfer element is fully compatible with existing distributed plasticity elements because both elements have the same mesh.

In the present work, the fiber heat transfer element is used in conjunction with a 2D force-based frame element adapted by the authors for structural fire simulation [14]. Previous studies [13,14] demonstrated that the fiber heat transfer and structural