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# Reactor similarity for plasma–material interactions in scaled-down tokamaks as the basis for the Vulcan conceptual design

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## ABSTRACT

Dimensionless parameter scaling techniques are a powerful tool in the study of complex physical systems, especially in tokamak fusion experiments where the cost of full-size devices is high. It is proposed that dimensionless similarity be used to study in a small-scale device the coupled issues of the scrape-off layer (SOL) plasma, plasma–material interactions (PMI), and the plasma-facing material (PFM) response expected in a tokamak fusion reactor. Complete similarity is not possible in a reduced-size device. In addition, "hard" technological limits on the achievable magnetic field and peak heat flux, as well as the necessity to produce non-inductive scenarios, must be taken into account. A practical approach is advocated, in which the most important dimensionless parameters are matched to a reactor in the reduced-size device, while relaxing those parameters which are far from a threshold in behavior. "Hard" technological limits are avoided, so that the reduced-size device is technologically feasible. A criticism on these grounds is offered of the "$P/R$" model, in which the ratio of power crossing the last closed flux surface (LCFS), $P$, to the device major radius, $R$, is held constant. A new set of scaling rules, referred to as the "$P/S$" scaling (where $S$ is the LCFS area) or the "PMI" scaling, is proposed: (i) non-inductive, steady-state operation; (ii) $P$ is scaled with $R^2$ so that LCFS areal power flux $P/S$ is constant; (iii) magnetic field $B$ constant; (iv) geometry (elongation, safety factor $q_*$, etc.) constant; (v) volume-averaged core density scaled as $n \approx \overline{n}_e \sim R^{-2/7}$; and (vi) ambient wall material temperature $T_{W,0}$ constant. It is shown that these scaling rules provide fidelity to reactor conditions in the divertor of the reduced-size device, allowing for reliable extrapolation of the behavior of the coupled SOL/PMI/PFM system from the reduced-size device to a reactor. The $P/S$ scaling is used as the basis for the Vulcan tokamak conceptual design.

## 1. Introduction and overview

The purpose of the Vulcan design study [1] is to determine if a small-scale experimental device can reasonably assess the complex and coupled boundary and PMI issues of future magnetic fusion reactors. In this paper, we argue that a similarity approach, in which key dimensionless parameters are kept as close as possible to reference values, is appropriate to guide the effort.

### 1.1. *The Vulcan conceptual design*

Vulcan is a conceptual design for a compact, high-field, steady-state tokamak that is intended to explore reactor-relevant plasma–material interaction (PMI) science issues such as hydrogenic fuel retention, material erosion, and heat exhaust to the first wall and divertor. In order for Vulcan to properly study the effect of the plasma on the materials and vice versa, the device must operate for long time scales ($\gtrsim$ months). An overview of Vulcan is given in a separate paper [1], while its key parameters are listed in Table 1.

### 1.2. *Overview of this paper*

The remainder of this work is laid out as follows. In Section 2, the utility and limitations of similarity for core and boundary fusion plasmas are generally considered. It is argued that both the physical reality of the eventual fusion reactor and its technology limits must be considered when making decisions about similarity in size scaling. In Section 3 we develop a new, extended set of dimensionless parameters required for similarity in PMI, from which it is shown that complete similarity is not possible in Vulcan or any scaled-down device. Section 4 then describes a "$P/S$"

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**Table 1**
Plasma parameters for three operational scenarios in Vulcan. The "A" scenario is full Vulcan performance; the "B" and "C" scenarios are more conservative for the first wall and for the current drive system, respectively. "Fraction of reactor-similarity density" refers to the ratio of the operating scenario electron density to the electron density required to create a completely reactor-like SOL according to the $P/S$ scaling rules.

| Parameter | Scenario | | |
|---|---|---|---|
| | A | B | C |
| Plasma major radius ($R$) (m) | 1.20 | 1.20 | 1.20 |
| Plasma aspect ratio ($R/a$) | 4.0 | 4.0 | 4.0 |
| Plasma elongation ($\kappa_{95}$) | 1.7 | 1.6 | 1.6 |
| Plasma triangularity ($\delta_{95}$) | 0.7 | 0.7 | 0.7 |
| LCFS area ($S$) ($m^2$) | 19.8 | 19.0 | 19.0 |
| Plasma volume inside LCFS ($m^3$) | 3.6 | 3.4 | 3.4 |
| LCFS power flux ($P/S$) ($MW\,m^{-2}$) | 1.0 | 0.75 | 1.0 |
| Total heating + CD power ($P_{ext}$) (MW) | 19.8 | 14.2 | 19.0 |
| On-axis magnetic field ($B_0$) (T) | 7.0 | 7.0 | 7.0 |
| Assumed $H$-factor ($H_{98}$) | 1.2 | 1.0 | 1.0 |
| Energy confinement time ($\tau_e$) (ms) | 94 | 61 | 35 |
| Vol.-avg. electron temp. ($\overline{T}_e$) (keV) | 2.7 | 1.6 | 3.0 |
| Fraction of reactor-similarity density | 1.0 | 0.75 | 0.30 |
| Vol.-avg. electron density ($\overline{n}_e$) ($10^{20}\,m^{-3}$) | 3.95 | 3.22 | 1.38 |
| Safety factor ($q_*$/equivalent $q_{cyl}$) | 3.0/1.54 | 4.0/2.25 | 4.0/2.25 |
| Total plasma current ($I_p$) (MA) | 1.70 | 1.17 | 1.17 |
| Assumed plasma charge state ($Z_{eff}$) | 1.3 | 1.3 | 1.3 |

similarity model which matches critical dimensionless parameters in the coupled scrape-off layer (SOL)/plasma–material interaction (PMI)/plasma-facing-material (PFM) system, and relaxes others, based on the observations of the previous two sections. Some concluding remarks are made in Section 5.

## 2. Derivation and comments on scrape-off layer similarity

### 2.1. Dimensionless scaling in tokamaks

Dimensionless parameter scaling techniques are a powerful tool in the study of complex physical systems. One seeks an appropriate set of controlling dimensionless parameters that provide similarity between experiments at different scales. This is particularly attractive in magnetic fusion research since reactors are large (plasma major radius $R \gtrsim 5$ m) and their cost high (cost $\gtrsim \$10^9$; core cost $\sim R^{2-3}$ at fixed volumetric power density). Kadomtsev proposed a set of eight dimensionless parameters controlling the tokamak core plasma [2], which has led to a rich field of core tokamak transport studies based on dimensionless scaling laws [3].

Though the work presented in this paper concentrates on the *boundary* plasma, it is illustrative and educational to examine the limitations and successes of this approach to *core* transport, i.e. what lessons have been learned from this large body of work. First, the number of possible combinations of dimensionless parameters is so large that one must use physical reasoning to prescribe the controlling parameters. In addition to obvious geometry similarity (aspect ratio $\equiv R/a = \epsilon^{-1}$, magnetic winding safety factor $q \propto \epsilon B_t/B_p$), the controlling dimensionless parameters are assumed to be set by density ($n$), temperature ($T$), magnetic field ($B$) and linear size ($R$), namely: normalized pressure $\beta \propto n T B^{-2}$, normalized collisionality $\nu^* \propto n R T^{-2}$, and normalized gyroradius $\rho^* \propto T^{1/2} R^{-1} B^{-1}$. Keeping these three dimensionless parameters constant leads to scaling laws based on linear size: $n \sim R^{-2}$, $T \sim R^{-1/2}$, and $B \sim R^{-5/4}$.

However, at this point, one must consider two important "realities" of magnetic fusion. First, of the parameters discussed above ($n$, $T$, $B$, and $R$), the magnetic field $B$ has a "hard" engineering limit due to the constraints on the field strength at the inner leg of the superconducting toroidal field coils. Second, in a fusion reactor at fixed $\epsilon$, economics force the designer to choose the highest $B$ field possible, since fusion power density $S_f \propto \beta^2 B^4$. Therefore, it will not be possible to implement the $B \sim R^{-5/4}$ scaling in a reduced-size model of a fusion reactor, and one must choose one or more dimensionless parameters to allow to vary.

The best choice is to allow $\rho^*$ to vary, informed again by physical reasoning: it is undesirable to change a dimensionless parameter near a threshold which could fundamentally change the system's behavior. In this case, the reactor $\beta$ will likely be near a stability limit (again, in order to maximize $S_f$), and varying $\nu^*$ can shift the system to a different transport regime, and change other neoclassical effects. In contrast, $\rho^*$ is always significantly less than unity, and is not expected to pass through any threshold, even in a scaled experiment. In addition, small-scale experiments can match a reactor's shape, $\beta$, and $\nu^*$ simultaneously, but not $\rho^*$. Most importantly, with this type of "practical" approach, the validity of the dimensionless similarity can be experimentally tested on present small-scale experiments. A large set of such experiments has strongly validated the use of similarity in understanding core plasma transport [3].

In this paper, we argue that similarity be applied to the coupled issues of the boundary (SOL) plasma, plasma–material interactions (PMI), and plasma-facing material (PFM) response, and that, generally, a "practical" approach be adopted such that small-scale devices can be used to gain insight into the issues facing a full-scale reactor. Because of the 2D nature of the boundary, and the added complexity of SOL/PMI/PFM science, the set of dimensionless parameters becomes much larger than in the core. However, three simple observations arise, again due to considerations of the reality of fusion reactors:

(i) Reactors must operate continuously for at least a year;
(ii) The steady-state heat flux incident on actively cooled surfaces has a hard technological limit set by material considerations, leading to a hard limit on $q_\parallel$, the parallel SOL heat flux at fixed geometry; and
(iii) In a fusion reactor, economics typically requires the highest areal fusion power density possible, which simultaneously maximizes the requirements for exhaust power handling ($\approx$1/5 of fusion power in D–T), including maximizing the heat flux incident on PFMs.

Note that (ii) and (iii) above are quite analogous to the restrictions on $B$ and $S_f$ for core similarity, while (i) is not considered whatsoever in core similarity arguments (i.e. the plasma system is always assumed to be stationary). However, stationary conditions cannot be assumed for the SOL/PMI/PFM problem: the PMI/PFMs will evolve continuously through processes such as erosion and fuel retention, and this evolution can couple back to the SOL conditions; for example, through time-varying impurity release.

### 2.2. Comments on the P/R model

Several studies in the 1990s (e.g. [4–6]) examined the expanded set of dimensionless parameters for boundary plasma similarity. Note that these studies only considered plasma properties (with a particular emphasis on simulating ITER boundary plasmas); PMI was not directly considered. It is beyond the scope of this paper to review these studies in detail. Nevertheless, it is illuminating to examine a simplified, explicit derivation of the "$P/R$" model of Lackner [4], where the main assertion was that similarity in atomic physics processes must be included in any model that seeks to provide similarity in the boundary plasma.

For this reason, another dimensionless parameter besides those stated by Kadomtsev, $T/E_{atomic}$, must be matched, where $T$ is plasma temperature and $E_{atomic}$ is a characteristic atomic energy, such