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Time–frequency analysis of fusion plasma signals beyond the short-time Fourier transform paradigm: An overview

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Abstract

Performing a time–frequency (*t*–*f*) analysis on actual magnetic pick-up coil data from the JET tokamak, a comparison is presented between the spectrogram and the Wigner and Choi–Williams distributions. Whereas the former, which stems from the short-time Fourier transform and has been the work-horse for *t*–*f* signal processing, implies an unavoidable trade-off between time and frequency resolutions, the latter two belong to a later generation of distributions that yield better, if not optimal joint *t*–*f* localization. Topics addressed include signal representation in the *t*–*f* plane, frequency identification and evolution, instantaneous-frequency estimation, and amplitude tracking.

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1. Introduction

The need for time–frequency (*t*–*f*) analysis [1,2] in nuclear fusion research, when processing data coming from dedicated high-temperature plasma diagnostics, has always been strongly felt [3–20]. Indeed, fusion plasmas are an incredibly rich source of physical phenomenology whose spectral content changes in time, as often seen in plasma turbulence and MHD instabilities. This gives rise to signals exhibiting, in general, time-varying spectra that can range from a few to hundreds of kHz, and can change as fast as in some hundredths of ms. The basic, almost universal approach to process and analyse such signals has been the short-time Fourier transform (STFT), from which the spectrogram (SPEC) has emerged as the ancestor of modern *t*–*f* distributions [1–11,13,16–20]. Sometimes, more sophisticated versions of time-local Fourier spectra, like wavelets [12,17], or alternative yet equivalent techniques, such as complex demodulation [14], have been used, but always within a framework that can be called the STFT paradigm. Despite their fair amount of success, the STFT and assimilated techniques sometimes fail to satisfactorily resolve in frequency rapid plasma events, as they are hindered by the well-known signal duration–bandwidth product [1,2,13,14,16,17,19]. To circumvent this limitation, new signal processing tools that are free of the trade-off between time and frequency resolutions, and of which the Wigner distribution (WD) is the most celebrated one [1,2,21,22], have made their appearance in fusion research [13,15–19].

From a historical perspective, the WD was first introduced in physics to deal with the quantum corrections to thermodynamic equilibrium [21], and later appeared in signal processing associated with the analytic signal [22]. Subsequently, a long-standing problem that remained to be satisfactorily solved, but has presently found its closure, was the proper definition of a WD for the discrete spectra of angular momentum in quantum mechanics [23], or for discrete-time (DT) signals in *t*–*f* analysis [19,24]. Still regarding data processing, and because the WD sometimes gives birth to artefacts in the *t*–*f* plane that arise from cross-term interference between different signal components, reduced-interference tools have been very much in demand, such as the Choi–Williams distribution (CWD) [1,2,25]. Both of these

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issues, namely, suitable use of DT WD forms when processing digitized data instead of continuous-time (CT) signals and reduction of deleterious cross-term effects via the CWD, have already been addressed for the purposes of fusion applications [16–19].

So, the aim of this paper is to give a critical comparison, as far as *t–f* analysis is concerned, between the performance of STFT techniques, such as the SPEC, and of a later generation of *t–f* tools, like the WD and the CWD. The analysis is carried out for signals coming from an actual tokamak diagnostic, a JET magnetic pick-up coil, and addresses several aspects of *t–f* analysis, including frequency identification and evolution, instantaneous-frequency (IF) estimation, and amplitude tracking. Not surprisingly, this overview draws essentially from the authors' own work [13–20], to which readers are referred for details on the expressions and techniques applied below, and also for a more complete list of the related literature. The authors' basic motivation has been the divulgation of these less-known *t–f* tools to as broader a nuclear fusion audience as possible, so fellow researchers can get well acquainted with and take full advantage of them. The worthiness of such a task can be easily justified by the significant fraction of reports monitoring the recent progress on controlled thermonuclear research in which *t–f* analysis is actually present, although always within the realm of the STFT paradigm [6–12].

2. Analysis

The signal to be processed comes from a magnetic pick-up coil operating during JET pulse #53060, and has been acquired at a rate $f_s = 250\,\text{kHz}$. The actual time and frequency variables are linked to the sample number n and the reduced frequency θ according to $n = tf_s$ and $\theta = 2\pi f/f_s$. Moreover, to get a proper complex signal, for the purposes of amplitude and IF retrieval, and also to avoid aliasing problems, the analytic signal $z(n)$ has been computed. The DT SPEC, WD, and CWD expressions read, respectively[3]:

$$P(n,\theta) = \left| \frac{1}{\sqrt{2\pi}} \sum_{m=-(l-1)/2}^{+(l-1)/2} z(n+m) w_l(m) \mathrm{e}^{-\mathrm{i}m\theta} \right|^2,$$

$$W(n,\theta) = \frac{1}{\pi} \sum_{m=-(l-1)/2}^{+(l-1)/2} z(n+m) z^*(n-m) w_l^2(m) \mathrm{e}^{-\mathrm{i}2m\theta},$$

and

$$\mathrm{CW}(n,\theta;\sigma) = \frac{1}{\pi} \sum_{m=-(l-1)/2}^{+(l-1)/2} \sum_{m'=-(l'-1)/2}^{+(l'-1)/2} z(n+m'+m) \times z^*(n+m'-m) \times w_l(m) h_{l'}(m') I(m',m;\sigma) \mathrm{e}^{-\mathrm{i}2m\theta},$$

with

$$I(m',m;\sigma) = \frac{\mathrm{e}^{-(m'/2m)^2\sigma}}{2|m|\sqrt{\pi/\sigma}}.$$

In the above equations, $w_l(m)$ and $h_{l'}(m')$ are real-valued, symmetric windows with an odd number of points l and l', respectively, the former taken here to be of the Hanning type and the latter a rectangular one, whereas $I(m',m;\sigma)$ is basically a Gaussian envelope whose width is controlled by the parameter σ. In what follows, $l = 1023$ for the SPEC and $l = 4095$ for the WD and CWD, $l' = 1023$, and $\sigma = 10$. As far as the WD and CWD are concerned, the windows $w_l(m)$ and $h_{l'}(m')$ are eminently introduced for practical, computational purposes and, provided they are sufficiently wide, have virtually no influence on neither time nor frequency resolution. Regarding σ, when it

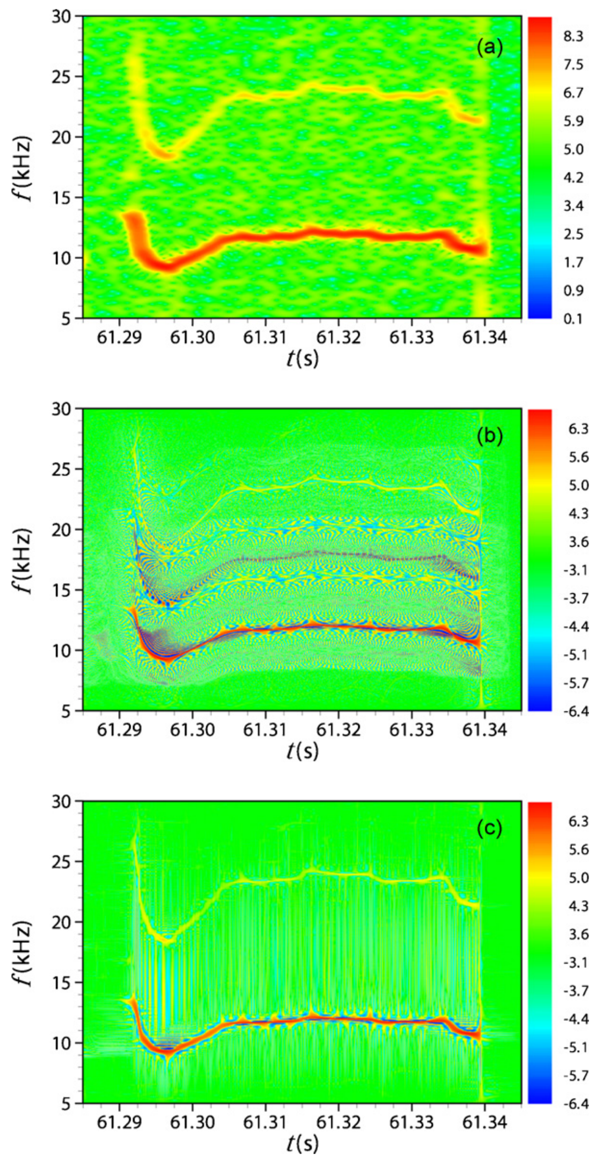

Fig. 1. Signal *t–f* representations obtained using $P(n,\theta)$, $W(n,\theta)$, and $\mathrm{CW}(n,\theta;\sigma)$, respectively, in (a), (b), and (c). For a given distribution $D(n,\theta)$, a logarithmic scale is used in the form $\pm\log[\pm D(n,\theta)]$ to account for the fact that $W(n,\theta)$ and $\mathrm{CW}(n,\theta;\sigma)$ can take negative values [16,17,19].

[3] Note that the WD and CWD forms given here have half the period of the signal's Fourier spectrum, in which case aliasing is avoided by using the analytic signal [13,16,19,24,25].