Geomechanics for Energy and the Environment 4 (2015) 29–38

Contents lists available at ScienceDirect

Geomechanics for Energy and the Environment

journal homepage: www.elsevier.com/locate/gete

Elastoplastic coupling for thermo-elasto-plasticity at high temperature

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HIGHLIGHTS

- Coupling of elastic properties with plastic strains, under non-isothermal conditions.
- Adiabatic elasto-plastic moduli.
- Refractory materials subjected to extreme temperature and stress conditions.
- Degradation of elastic stiffness due to plastic distorsional strains.
- Variation of elastic stiffness due to plastic volumetric compaction/dilation.

ARTICLE INFO

Article history:
Received 8 June 2015
Received in revised form 22 September 2015
Accepted 22 September 2015
Available online 8 October 2015

Dedicated to Prof. Tomasz Hueckel, on the occasion of his 70th birthday

Keywords:
Elasticity
Plasticity
Flow rule

ABSTRACT

The coupling of elastic and plastic deformation is introduced in a framework allowing for the inclusion of thermal effects. In this context where high temperatures may be present, many laws governing materials behaviour become nonlinear functions of both temperature and strain. The understanding of this is crucial to model the behaviour of refractories, sintering processes, rocks at great depth and geo-energy. Two equivalent formulations are presented, one in terms of isothermal elastic quantities and the other in terms of adiabatic. In the former approach a Helmholtz free energy density need not be introduced and the contributions to the variations of elastic stiffness of both plastic flow and temperature can be clearly separated and hence a full explanation is given. A simple example using available experiments on sintered alumina shows the capability of the constitutive framework to describe the behaviour of refractories at high temperature.

1. Introduction

Refractories for applications in the liquid steel industry,[1] ceramic powders subject to sintering,[2,3] rocks in the earth's mantle, rocks involved in heat storage or in geothermal energy exploitation at great depth,[4] and concrete for fire protection[5] are all examples of materials working under extreme temperature environments or subject to high temperature excursions. In all these cases plastic flow which is caused by sliding between grains, nucleation and growth of pores and microcracks affects the elastic properties of the materials. These phenomena can be modelled using the concept of elastoplastic coupling, a concept that was pioneered and developed by Tomasz Hueckel,[6–10] to whom this article is dedicated. In the above-mentioned materials, temperature is known to influence yielding, hardening, and elasticity, but it may have antagonistic effects on damage, so that, while a thermal shock can microfracture a brittle material, adhesion between particles occurs during sintering and an analogous phenomenon of microfracture healing may happen as a consequence of local melting at crack surfaces in fissured rock.

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http://dx.doi.org/10.1016/j.gete.2015.09.001
2352-3808/

Research on elastoplastic coupling has developed to include multisurface effects,[11] models for the densification processes of granulates[12–15] and models for the mechanical response of sand.[16] This has proved to be of crucial importance in capturing strain localization in granular media under drained conditions.[17,18] An important step forward in this activity was the realization that assuming normality for the rate of irreversible strain (rather than of plastic strain) to the yield surface implies that the constitutive operator is symmetric,[17] an observation which becomes crucial in the definition of maximum dissipation.

The present article is focused on a class of materials displaying only reversible strain when subjected to thermal load/unload cycles, with the stress state lying within the yield surface. This reversibility is typical of single-constituent refractory materials (such as sintered alumina[19,20]) and implies that a temperature variation may produce plastic strain only if the stress state lies on the yield surface. This behaviour is in contrast with materials (made up with various constituents of different thermal expansion coefficients[21]) displaying irreversible strains when subjected to thermal load/unload cycles at null applied stresses, thus involving a complex coupling between plastic strain and temperature. The purpose of the present article is to develop the formulation of elastoplastic coupling for the modelling of thermoplastic behaviour of rock-like and refractory materials. The main complication here arises from the fact that several constitutive laws, usually assumed to be linear, become nonlinear for high temperature excursions and functions of both plastic strain (which measures the damage of the material) and temperature. Therefore, following [22] the formulation will first be introduced as a direct extension of elastoplastic coupling under isothermal conditions (Section 2.1), which does not require the introduction of a free energy density function, and later transformed into an adiabatic form (Section 2.2). The treatment is done in such a way that the contributions of elastoplastic coupling and of temperature on variation of elastic stiffness are clearly separated. This separation is crucial to describe the sometimes contrasting effects of damage (reducing the elastic stiffness) and temperature (which, when extreme, may induce melting and damage healing). Finally, an application is presented to describe the sintering of a ceramic powder or refractory material in which the self-healing occurs with exposure to high temperature. (Section 3). The example is calibrated using experimental data on sintered alumina and shows the capability of the constitutive framework for the description of rock-like materials at high temperature.

2. Thermoplastic constitutive equations

Two formulations are introduced for thermo-elastoplastic constitutive equations in which the plastic deformation influences the elastic response of the material ('elastoplastic coupling'), with a full account of all couplings related to the temperature effects. The first formulation, obtained as a direct extension of the elastoplastic coupling concept under isothermal conditions, does not require the introduction of a Helmholtz free energy density, while the second formulation does require its introduction, based on adiabatic quantities.

2.1. Formulation in isothermal form

The strain $\boldsymbol{\epsilon}$ is additively decomposed into the elastic (subscript 'e'), plastic (subscript 'p'), and thermal (subscript 'T') parts as

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_e + \boldsymbol{\epsilon}_p + \boldsymbol{\epsilon}_T, \tag{1}$$

where the thermal strain $\boldsymbol{\epsilon}_T$ can be a complicated tensorial function of the temperature variation ΔT. In the case of isotropic response, the thermal strain is equal to

$$\boldsymbol{\epsilon}_T = \alpha_{(T)} \Delta T \, \mathbf{I} \tag{2}$$

where the thermal expansion coefficient $\alpha_{(T)}$ can be constant or assumed to be a function of the temperature.

The stress can be expressed as a function of the strain, of its plastic part, and of the temperature

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}\left(\boldsymbol{\epsilon}, \boldsymbol{\epsilon}_p, T\right), \tag{3}$$

implying a precise choice of independent variables. For instance, the stress can be a linear relation of the elastic strain through a fourth-order elastic tensor $\mathbb{E}_i$, which may depend on both temperature and plastic strain, namely, $\mathbb{E}_i(\boldsymbol{\epsilon}_p, T)$, so that

$$\boldsymbol{\sigma} = \mathbb{E}_{i\,(\boldsymbol{\epsilon}_p,T)}[\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_p] + \Delta T \mathbf{C}_{(\boldsymbol{\epsilon}_p,T)}. \tag{4}$$

The subscript '$(\boldsymbol{\epsilon}_p, T)$' highlights the functional dependence of the quantity on the plastic strain and the temperature. In Eq. (4), $\mathbf{C}$ is a second-order symmetric tensor, that, in the particular case of isotropic thermal strain and isotropic elastic bulk response, reduces to

$$\Delta T \mathbf{C}_{(\boldsymbol{\epsilon}_p,T)} = -\mathbb{E}_{i\,(\boldsymbol{\epsilon}_p,T)}[\boldsymbol{\epsilon}_T] = -3K_{i\,(\boldsymbol{\epsilon}_p,T)}\alpha_{(T)}\Delta T\,\mathbf{I}, \tag{5}$$

where $K_{i\,(\boldsymbol{\epsilon}_p,T)}$ is the elastic isothermal bulk modulus, which may be function of both the plastic strain and the temperature, $K_i(\boldsymbol{\epsilon}_p, T)$.

The stress–temperature tensor is defined as

$$\mathbf{B} = \frac{\partial \boldsymbol{\sigma}(\boldsymbol{\epsilon}, \boldsymbol{\epsilon}_p, T)}{\partial T}, \tag{6}$$

so that, accepting representation (4) becomes

$$\mathbf{B} = \frac{\partial \mathbb{E}_{i\,(\boldsymbol{\epsilon}_p,T)}}{\partial T}[\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_p] + \Delta T \frac{\partial \mathbf{C}_{(\boldsymbol{\epsilon}_p,T)}}{\partial T} + \mathbf{C}_{(\boldsymbol{\epsilon}_p,T)}. \tag{7}$$

It can be noted that, for the simplified law (5), the derivative of $\mathbf{C}$ with respect to the temperature reduces to

$$\Delta T \frac{\partial \mathbf{C}_{(\boldsymbol{\epsilon}_p,T)}}{\partial T} = -3\Delta T \frac{\partial K_{i\,(\boldsymbol{\epsilon}_p,T)}}{\partial T}\alpha_{(T)}\,\mathbf{I} - 3\Delta T K_{i\,(\boldsymbol{\epsilon}_p,T)} \frac{\partial \alpha_{(T)}}{\partial T}\,\mathbf{I}, \tag{8}$$

which, for constant $\alpha_{(T)}$, is only a measure of the temperature variation of the elastic stiffness.

Taking the derivative of Eq. (4) with respect to the time, and accepting elastoplastic coupling (so that the dependence of $\mathbb{E}_{i\,(\boldsymbol{\epsilon}_p,T)}$ on the plastic strain is not neglected), the stress rate can be expressed as

$$\dot{\boldsymbol{\sigma}} = \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\epsilon}}[\dot{\boldsymbol{\epsilon}}] + \frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\epsilon}_p}[\dot{\boldsymbol{\epsilon}}_p] + \mathbf{B}\dot{T}, \tag{9}$$