Geotextiles and Geomembranes 44 (2016) 122–131

Contents lists available at ScienceDirect

Geotextiles and Geomembranes

journal homepage: www.elsevier.com/locate/geotexmem

Technical note

Vertical-drain consolidation using stone columns: An analytical solution with an impeded drainage boundary under multi-ramp loading

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ARTICLE INFO

Article history:
Received 8 December 2014
Received in revised form
22 June 2015
Accepted 3 July 2015
Available online 22 July 2015

Keywords:
Consolidation
Pore pressures
Ground improvement
Embankments

ABSTRACT

An analytical solution is derived to predict consolidation with vertical drains under impeded drainage boundary conditions and multi-ramp surcharge loading. The impeded drainage is modelled by adopting the third type boundary condition with a dimensionless characteristic factor of drainage efficiency developed by Gray (1945) for one-dimensional consolidation. Fully drained and undrained boundary conditions can also be modelled by applying an infinite and a zero characteristic factor, respectively. The combined effects of drain resistance and smear are taken into account fully. An explicit, rigorous analytical solution is derived using the method of separation of variables to calculate excess pore-water pressure at any arbitrary point in soil and to derive the overall average degree of consolidation. The proposed solution can also be used to analyse one-dimensional consolidation without vertical drains but with an impeded drainage boundary. Its validity and accuracy are verified by comparing the proposed solution with the solutions developed by Gray (1945) and Terzaghi (1943). Its practical applicability is also evaluated by analysing a case history involving a fill embankment, which was constructed over a crust layer of hard soil overlying soft clay improved with stone columns. The crust layer is modelled as an impeded drainage. Reasonably good agreement is obtained between the consolidation results obtained from the proposed analytical solution and available three-dimensional finite-element predictions. With the further consideration of smear effects, good agreement is achieved between the consolidation results obtained from the proposed analytical solution and field measurements.

1. Introduction

Soft soil is often preloaded with surcharge pressure as one of the most economic and effective ways to consolidate it (Qubain et al., 2014). Vertical prefabricated drains or sand/stone columns are commonly utilised to accelerate the consolidation of soft soils under preloading (Almeida et al., 2015; Artidteang et al., 2011; Cascone and Biondi, 2013; Chai et al., 2010; Indraratna et al., 2010; Jang and Chung, 2014; Karunaratne, 2011; Li and Rowe, 2001; Lo et al., 2008, 2010; Rowe and Li, 2005; Rowe and Taechakumthorn, 2008; Saowapakpiboon et al., 2009, 2010; Shen et al., 2005; Suleiman et al., 2014; Van Helden et al., 2008; Voottipruex et al., 2014; Xue et al., 2014). Analytical solutions predicting the extent of consolidation in preloading play an important role in the preliminary design of vertical drains (Abuel-Naga et al., 2012; Bari and Shahin, 2014; Basu and Prezzi, 2009; Chung et al., 2014; Rujikiatkamjorn and Indraratna, 2009; Sinha et al., 2009). Since the pioneering work of Barron (1948), the challenge of deriving an analytical solution for cylindrical unit-cell consolidation with a vertical drain has captured the attention of the ground improvement community. For consolidation of a single layer of homogeneous soil under surcharge preloading, various solutions have been proposed based on different assumptions and considerations. A large number of analytical solutions were derived for the consolidation of soil with fully drained boundary conditions at its top and/or bottom surface (e.g., Conte and Troncone, 2009; Deng et al., 2013a, 2013b; Indraratna et al., 2011; Kianfar et al., 2013; Lei et al., 2015; Lu et al., 2011, 2015; Ong et al., 2012;

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http://dx.doi.org/10.1016/j.geotexmem.2015.07.003
0266-1144/© 2015 Published by Elsevier Ltd.

Rujikiatkamjorn and Indraratna, 2009, 2015; Walker et al., 2012; Zwanenburg and Barends, 2006). However, in reality, impeded drainage is a matter of concern to practising engineers when the permeability of drainage medium at a boundary surface is not high enough to discharge the water expelled from the consolidating soil promptly (Chai and Miura, 1999; Duncan, 1993; Olson, 1998). This may occur when the consolidating soil is overlain by an improperly graded sand blanket, which is prone to fouling and clogging by infiltration of fine particles due to consolidation and by intrusion of fine particles due to fill compaction. Its hydraulic conductivity may be significantly reduced by fine content (Andersen and Schjetne, 2013; Bandini and Sathiskumar, 2009; Duong et al., 2014; Tennakoon et al., 2012). Under these circumstances, the hydraulic boundary condition would be more appropriately modelled as an impeded drainage.

According to the continuity of the excess pore-water pressure and the flow rate at the interface between the consolidating soil and its surface drainage medium, Gray (1945) showed that an impeded drainage can be described by the third type boundary condition as follows:

$$\frac{\partial u_b}{\partial z} = \pm R\frac{u_b}{h} \quad \text{with} \quad R = \frac{k_{vi}h}{k_v h_i} \tag{1}$$

where z is the vertical coordinate directed downwards; accordingly, the signs '+' and '−' in Eq. (1) are adopted for the top and bottom boundaries of the consolidating soil, respectively; u_b is the excess pore-water pressure at the boundary; h is the thickness of the consolidating soil layer; R is a dimensionless characteristic factor of drainage efficiency; k_v and k_{vi} are the vertical hydraulic conductivity of the consolidating soil and its surface drainage medium, respectively; and h_i is the thickness of the drainage medium. The fully drained and undrained boundary conditions can also be described by Eq. (1) with $R = \infty$ and $R = 0$, respectively.

Based on the hydraulic boundary condition given by Eq. (1), several analytical solutions have been proposed for solving one-dimensional consolidation problems (Gray, 1945; Mesri, 1973; Schiffman and Stein, 1970; Xie et al., 1999). For cylindrical unit-cell consolidation with a vertical drain, the existing analytical solutions were derived only under instantaneous loading conditions (Cheng et al., 2003; Liu et al., 2007; Sun et al., 2007; Zhang et al., 2005). However, in practical situations, surcharge loading is almost always gradually and incrementally applied. The total stress in soil increases synchronously with the increase in surcharge loading. Such loading conditions would be more appropriately modelled as a time-dependent increase in total stress under multi-ramp loading.

An explicit, rigorous analytical solution is derived herein for equal-strain consolidation with a vertical drain under impeded drainage boundary conditions and multi-ramp loading. The excess pore-water pressure at any arbitrary point in soil is obtained, together with the overall average degree of consolidation. The proposed solution can also be used to analyse vertical-drain consolidation under fully drained conditions and one-dimensional consolidation under impeded drainage boundary conditions.

2. Problem description

Fig. 1 shows a unit-cell model for consolidation of undisturbed and smeared soils with a vertical drain. The soils are subjected to a time-dependent increase in total stress under multi-ramp loading. The governing equations of equal-strain consolidation of undisturbed and smeared soils assuming constant material properties (Terzaghi, 1943; Barron, 1948) are given in full by

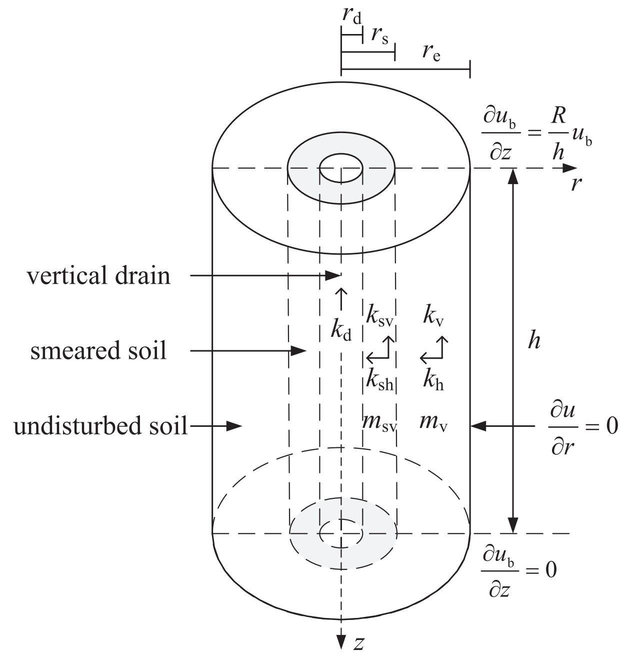

Fig. 1. A unit-cell model of consolidation under impeded drainage boundary conditions.

$$\frac{k_h}{\gamma_w}\left[\frac{\partial^2 u(r,z,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,z,t)}{\partial r}\right] + \frac{k_v}{\gamma_w}\frac{\partial^2 u(r,z,t)}{\partial z^2} = -m_v\left[\frac{\partial \sigma(t)}{\partial t} - \frac{\partial \overline{u}(z,t)}{\partial t}\right], \quad r_s \le r \le r_e \tag{2}$$

$$\frac{k_{sh}}{\gamma_w}\left[\frac{\partial^2 u_s(r,z,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u_s(r,z,t)}{\partial r}\right] + \frac{k_{sv}}{\gamma_w}\frac{\partial^2 u_s(r,z,t)}{\partial z^2} = -m_{sv}\left[\frac{\partial \sigma(t)}{\partial t} - \frac{\partial \overline{u}_s(z,t)}{\partial t}\right], \quad r_d \le r \le r_s \tag{3}$$

where r and z are the radial and vertical coordinates, respectively; t is time; r_d, r_s and r_e are the radii of the vertical drain, the smear zone and the effective influence zone of the vertical drain, respectively; u and u_s are the excess pore-water pressure of undisturbed soil and smeared soil, respectively; σ is the increase in total stress in soil due to surcharge loading; $\overline{u}$ and $\overline{u}_s$ are the average excess pore-water pressure at a given depth in the radial direction between r_s and r_e and between r_d and r_s, respectively; k_h, k_v and m_v are the horizontal and vertical hydraulic conductivity and volume compressibility of the undisturbed soil, respectively; k_{sh}, k_{sv} and m_{sv} are the horizontal and vertical hydraulic conductivity and volume compressibility of the smeared soil, respectively; and γ_w is the unit weight of water.

According to the continuity of the excess pore-water pressure and the flow rate at the interface between the vertical drain and the smeared soil, the drain resistance can be expressed as (Barron, 1948; Hansbo, 1981)

$$2k_{sh}\left(\frac{\partial u_s}{\partial r}\right) + r_d k_d\left(\frac{\partial^2 u_s}{\partial z^2}\right) = 0, \quad r = r_d \tag{4}$$

where k_d is the hydraulic conductivity of the vertical drain.

The continuity of the excess pore-water pressure and the flow rate at the interface between the smeared soil and the undisturbed soil can be described by

$$u = u_s, \quad r = r_s \tag{5}$$