Geotextiles and Geomembranes 43 (2015) 182–189

Contents lists available at ScienceDirect

Geotextiles and Geomembranes

journal homepage: www.elsevier.com/locate/geotexmem

Technical note

Nonlinear consolidation of vertical drains with coupled radial–vertical flow considering well resistance

Mengmeng Lu [a], [b], *, [1], Shanyong Wang [b], Scott W. Sloan [b], Daichao Sheng [b], Kanghe Xie [c]

[a] *State Key Laboratory for Geomechanics and Deep Underground Engineering, School of Mechanics & Civil Engineering, China University of Mining & Technology, Xuzhou 221116, China*

[b] *Australia Research Council Centre of Excellence for Geotechnical Science & Engineering, The University of Newcastle, Callaghan, NSW 2308, Australia*

[c] *Research Centre of Coastal and Urban Geotechnical Engineering, Zhejiang University, Hangzhou 310058, China*

ARTICLE INFO

Article history:
Received 1 July 2014
Received in revised form
25 November 2014
Accepted 16 December 2014
Available online 5 January 2015

Keywords:
Consolidation
Vertical drain
Nonlinearity
Well resistance
Coupled radial–vertical flow
Instantaneous loading
Single-stage loading
Multi-stage loading

ABSTRACT

The consolidation behavior of ground with vertical drains is known to be greatly affected by the finite permeability of the sand drains, also called the effect of well resistance. However, up to now, no analytical methods have been reported for evaluating this effect on the nonlinear consolidation behavior of vertical drains. In this paper, by considering the nonlinear compressibility and permeability of soil during consolidation, the effect of well resistance was incorporated into the derivation of the equations that govern the nonlinear consolidation of a vertical drain with coupled radial–vertical flow. In addition, the smear effect was considered by assuming three decay patterns for the radial permeability coefficients of the soil toward the sand drain in the smeared zone. After obtaining the governing equations, a simplified analytical solution is derived for a general time-variable surcharge loading. Based on the general solution obtained, detailed solutions are provided for three special types of loading schemes: constant loading, single-stage loading, and multi-stage loading. The validity of the solution is verified by reducing it to several special cases and comparing these to existing solutions. Finally, the effect of the well resistance, the ratios of the compression index to the radial and vertical permeability indices, various loading schemes, and various variation patterns of the radial permeability coefficient of the soil in the smeared soil zone are investigated using parametric analysis.

1. Introduction

Vertical drains, such as prefabricated vertical drains (PVDs) and sand drains, are used worldwide to accelerate the consolidation of soft soils. The first analytical study on the consolidation of vertical drains was performed by Barron (1948), where the water was assumed to flow radially into the sand drain and the vertical flow in the soil was ignored. After this study, many works, including both linear and nonlinear consolidation theories for vertical drains, were performed to extend this theory to reflect more realistic engineering problems.

The linear consolidation theory for vertical drains, which commonly assumes a constant permeability and compressibility during consolidation, has experienced great progress in the past few decades. In Barron's theory (1948), the surcharge loading was assumed to be fully applied instantaneously on the ground. Obviously, this instantaneous loading cannot be achieved in practice because the construction of superstructures requires time to be accomplished. In fact, the loading rate and the loading pattern have been proven to greatly influence the consolidation behavior of soils with vertical drains (Tang and Onitsuka, 2000). For this reason, many studies (Zhu and Yin, 2004; Leo, 2004; Conte and Troncone, 2009; Lu et al., 2011; Geng et al., 2012) were performed to account for some commonly used loadings, such as ramp loading, multi-stage loading, time- and depth-dependent loading, and vacuum-surcharge loading. Apart from the loading scheme, the smear effect is also known to influence the consolidation behavior of vertical drains. Walker and Indraratna (2006, 2007) studied the smear effect by considering a parabolic distribution of the permeability coefficient of the soil and the overlapping smear zone due to

---

* Corresponding author. State Key Laboratory for Geomechanics and Deep Underground Engineering, School of Mechanics & Civil Engineering, China University of Mining & Technology, Xuzhou 221116, China. Tel.: +86 15262026232; fax: +86 516 83590666.

E-mail address: lumm79@126.com (M. Lu).

[1] Tel.: +61 469362858; fax: +61 2 49215500.

http://dx.doi.org/10.1016/j.geotexmem.2014.12.001

the installation of sand-drains. Their studies provided a possible explanation for why a reduction in the size of the influence zone has a limited effect on accelerating the consolidation rate. In addition, Walker et al. (2009) numerically analyzed the consolidation of multi-layered vertical drains using a spectral method. However, this method is difficult for engineers to use due to its complex programming requirements. By introducing the double porosity model into the analysis of consolidation with vertical drains, Wang and Jiao (2004) provided a simplified approach for considering the smear effect of vertical drains, which has since been used by Lu et al. (2011, 2014). To develop an appropriate equation to convert the rectangular shape of a PVD to an equivalent circular shape, Abuel-Naga et al. (2012) assessed the validity of several existing approaches numerically and identified the most appropriate one for use in design. Incorporating the elastic visco-plastic constitutive equation of Yin and Graham (1989, 1994, 1996), Hu et al. (2014) suggested that including the large-strain condition and the creep effects was essential in the analysis of the consolidation of very soft clay layers with vertical drains. Compared to the aforementioned linear consolidation theories, fewer developments have been made for the nonlinear consolidation of vertical drains due to the complexity of the problem. For this reason, Lekha et al. (1998) proposed an approximate analytical method to investigate the nonlinear radial consolidation of vertical drains under ramp loading. Likewise, using the same nonlinear relation of a soil's compression and permeability with effective stress, Indraratna et al. (2005) solved the consolidation of vertical drains under instantaneous loading analytically and compared their results with data measured in the laboratory with good agreement found between the two sets of results. For this reason, Lu et al. (2014) used the same method to derive a simplified analytical solution for the nonlinear consolidation of vertical drains that are subjected to a general time-variable loading. In a different approach, Walker et al. (2012) analyzed the nonlinear consolidation of vertical drains numerically where a non-Darcian flow law was incorporated. However, this numerical method is difficult for engineers to use in an actual design.

Thus, although some progress has been made on the consolidation behavior of vertical drains, particularly for linear consolidation theory, no analytical solutions for the nonlinear consolidation of vertical drains allowing for the effect of well resistance are available. As Hansbo et al. (1981) noted, the effect of well resistance should be taken into account, especially when the drains are deeply installed. Well resistance in the field can be caused by several factors, such as the reduction in cross-sectional area of the PVDs, the deformation of the PVDs, and fine soil particles infiltrating into the PVD core and clogging the drainage channels (Deng et al., 2013). In this context, this paper addresses the problem by incorporating a finite permeability coefficient for a sand drain into the analysis of the consolidation of a vertical drain with coupled radial and vertical flow. In addition, to account for the unfavorable effects of installation on the adjacent soil, the decay of the radial permeability of the smeared soil towards a sand drain is also considered in the analysis.

2. Basic assumptions, objectives, equations, and solutions

The main objective of this paper is to develop an easy-to-use analytical solution for the nonlinear consolidation of a vertical drain with coupled radial and vertical flows by considering the finite permeability of the vertical drain (well resistance), the variation of the radial permeability of the soil in the smeared soil zone (the smear effect), and time-varying surcharge loading. The unit cell for analysis can be idealized as shown in Fig. 1, and the assumptions made in the analysis are listed below.

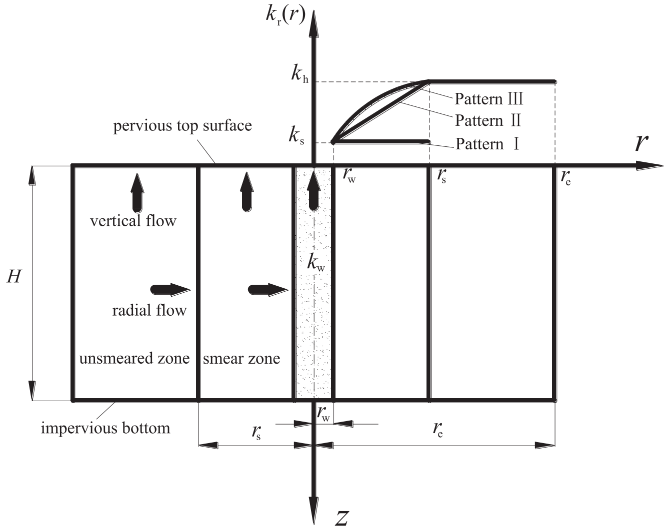

Fig. 1. Schematic diagram of the vertical drain showing the variation of the radial coefficient of soil permeability.

① The following relations are adopted for the nonlinear response of the soil permeability and compressibility during consolidation:

$$e = e_0 - C_c \log\frac{\bar{\sigma}'_s}{\bar{\sigma}'_{s0}} \tag{1.a}$$

$$e = e_0 + C_{kh} \log\frac{k_h}{k_{h0}} \tag{1.b}$$

$$e = e_0 + C_{kv} \log\frac{k_v}{k_{v0}} \tag{1.c}$$

where C_c, C_{kh}, and C_{kv} are the compression index, the radial permeability index, and the vertical permeability index, respectively; k_h and k_{h0} are the radial permeability coefficients of the unsmeared soil at any time and at the initial time, respectively; k_v and k_{v0} are the vertical permeability coefficients of the unsmeared soil at any time and at the initial time; e and e_0 are the void ratios of the soil at any time and at the initial time; and $\bar{\sigma}'_s$ and $\bar{\sigma}'_{s0}$ are the average effective stresses in the soil at any time and at the initial time.

From Eqs. (1.a)–(1.c), the following basic relations are obtained:

$$\frac{k_v}{k_{v0}} = \left(\frac{\bar{\sigma}'_s}{\bar{\sigma}'_{s0}}\right)^{-\frac{C_c}{C_{kv}}} \tag{2.a}$$

$$\frac{k_v}{k_h} = \frac{k_{v0}}{k_{h0}}\left(\frac{\bar{\sigma}'_s}{\bar{\sigma}'_{s0}}\right)^{\frac{C_c}{C_{kh}}-\frac{C_c}{C_{kv}}} \tag{2.b}$$

$$\frac{a_v}{a_{v0}} = \frac{(\partial\bar{e}/\partial\bar{\sigma}'_s)}{(\partial\bar{e}/\partial\bar{\sigma}'_s)_{t=0}} = \frac{\bar{\sigma}'_{s0}}{\bar{\sigma}'_s} = \left(\frac{\bar{\sigma}'_s}{\bar{\sigma}'_{s0}}\right)^{-1} \tag{2.c}$$

where a_v and a_{v0} are the compression coefficients of the soil at any time and at the initial time, respectively.

② During the consolidation process, the soil permeability reduces because the void ratio of soil diminishes as the