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Technical note

General solutions for consolidation of multilayered soil with a vertical drain system

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ABSTRACT

A quasi-analytical method is newly introduced to solve the equal-strain consolidation problem of multilayered soil with a vertical drain system. Both vertical and radial drainage conditions are considered, together with the effects of drain resistance and smear. By using the method of Laplace transform with respect to time, a general explicit analytical solution for the consolidation in transformed space is obtained. Numerical inversion of the Laplace transform in the time domain is then applied to obtain the solution for calculating excess pore-water pressure. This solution is explicitly expressed and conveniently coded into a computer program for ease and efficiency of practical use. Its validity and accuracy are verified by comparing the special cases of the proposed solution with a finite-element solution and an available analytical solution. Moreover, the consolidation behavior of a four-layered soil with a vertical drain is investigated. The order of soil layers is shown to have a significant effect on the behavior of consolidation. This highlights that caution should be exercised when weighted average consolidation parameters of multilayered soil are used to analyze the consolidation behavior.

1. Introduction

Preloading is widely used to improve soft ground, and vertical drains are usually installed to accelerate the consolidation of soils in preloading (Artidteang et al., 2011; Cascone and Biondi, 2013; Chai et al., 2008, 2010; Indraratna et al., 2010, 2011, 2012; Karunaratne, 2011; Lin and Chang, 2009; Lo et al., 2008; Saowapakpiboon et al., 2009, 2010, 2011). The shear strength of soil can be increased and the post-construction foundation settlement can be eliminated significantly due to consolidation (Almeida et al., 2013; Chai and Duy, 2013; Dash and Bora, 2013; Elsawy, 2013; Rowe and Li, 2005; Rowe and Taechakumthorn, 2008). For consolidation of a homogeneous soil with a vertical drain, many analytical solutions have been proposed based on various assumptions and considerations (Abuel-Naga et al., 2012; Bari and Shahin, 2014; Barron, 1948; Castro and Sagaseta, 2013; Chai et al., 2001; Conte and Troncone, 2009; Deng et al., 2013; Geng et al., 2011, 2012; Indraratna et al., 2011; Hansbo, 1981; Hu et al., 2014; Leo, 2004; Ong et al., 2012; Onoue, 1988b; Rujikiatkamjorn and Indraratna, 2009; Tang and Onitsuka, 2000; Yoshikuni and Nakanodo, 1974; Zeng and Xie, 1989; Zhu and Yin, 2001, 2004; among others). However, natural sediments are rarely homogeneous and usually consist of several different soil layers. For consolidation of layered soil with a vertical drain, only a limited number of analytical solutions are available in literature. Xie (1995) proposed a solution for consolidation of two-layered soil with an ideal drain (i.e., without drain resistance and smear actions) by using the method of separation of variables. Tang and Onitsuka (2001) and Wang and Jiao (2004) extended this solution to include drain resistance and smear effects. On this basis, Tang et al. (2013) have recently proposed a solution for consolidation of three-layered soil. Although the solution is precise, it has been proven to have some convergence problems (Tang et al., 2013). By simply using the method of separation of variables, it is difficult to extend the solution to consolidation of soils of four or more layers. For arbitrarily multilayered soil, Rujikiatkamjorn and Indraratna (2010) proposed a solution for purely radial consolidation based on the soil slice (instead of soil element) flow continuity equations, which are similar to that developed by Hansbo (1981). Vertical drainage conditions were neglected. Nogami and Li (2003) used the matrix

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transfer method to formulate the consolidation behavior of soil with a system of horizontal thin drains and a vertical drain. However, smear effect and drain resistance were not considered in their solution. Walker and Indraratna (2009) and Walker et al. (2009) used the spectral method to obtain a solution for consolidation of multilayered soil based on a lumped governing equation derived from the soil slice flow continuity, but drain resistance was ignored. Moreover, a transcendental equation is usually involved in the available analytical solutions for consolidation of layered soil. The most difficult part of the solutions is calculating, without missing, eigenvalues or eigenvectors from the zeros of transcendental equation.

In this note, a powerful Laplace transform and its numerical inverse technique is used to formulate the consolidation behavior of arbitrarily multilayered soil with a vertical drain. The assumptions and flow continuity conditions involved are same as those applied to two-layered soil by Tang and Onitsuka (2001) and three-layered soil by Tang et al. (2013). Radial and vertical drainage conditions, as well as drain resistance and smear effects, are considered. An explicit quasi-analytical solution is derived for calculating excess pore-water pressure and the degree of consolidation. The validity and accuracy of this solution are verified against a finite-element solution for two-layered soil, and an analytical solution derived for single-layered soil by Tang and Onitsuka (2000). Moreover, the consolidation behavior of a four-layered soil with a vertical drain system is investigated.

2. Problem description

The system consisting of N contiguous homogeneous soil layers with a vertical drain is shown schematically in Fig. 1. The origin of the cylindrical coordinates is set at the top center of the vertical drain; z is positive in the downward direction; and r is in the radial direction. In Fig. 1, r_w, r_s and r_e represent the radii of the vertical drain, the smear zone and the influence zone of the vertical drain, respectively; and k_w represents the vertical hydraulic conductivity of vertical drain. Although k_w varies with the consolidation time and the confining pressure around the vertical drain (Venda Oliveira, 2013), this is beyond the scope of this study. For the ith layer of soil, h_i, k_{hi}, k_{vi}, k_{si} and m_{vi} represent, respectively, the thickness, the horizontal and vertical hydraulic conductivity of natural soil, the horizontal hydraulic conductivity of smeared soil and the coefficient of volume compressibility of the soils. The external load $q(t)$ (where t is time) is assumed to be time dependent. The top surface of the system is assumed to be pervious, and the bottom surface is impervious.

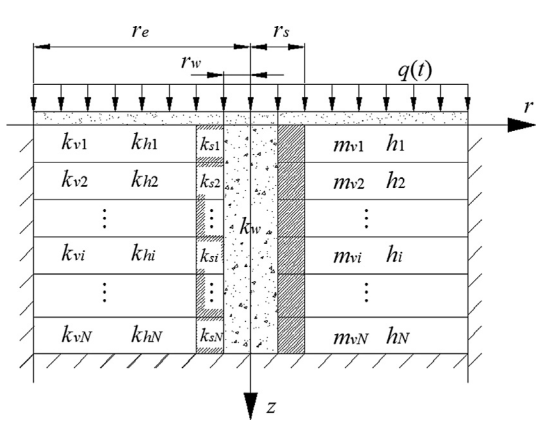

Fig. 1. Mathematical model of multilayered ground with vertical drain.

According to the assumptions made in Tang and Onitsuka (2000, 2001) and Tang et al. (2013), the following consolidation equations can be obtained.

Consolidation in the natural zone can be expressed as

$$\frac{k_{hi}}{\gamma_w}\left(\frac{1}{r}\frac{\partial u_{ni}}{\partial r}+\frac{\partial^2 u_{ni}}{\partial r^2}\right)+\frac{k_{vi}}{\gamma_w}\frac{\partial^2 \overline{u_i}}{\partial z^2}=m_{vi}\left(\frac{\partial \overline{u_i}}{\partial t}-\frac{\mathrm{d}q}{\mathrm{d}t}\right)\quad r_s\le r\le r_e \tag{1}$$

where $\gamma_w = 10$ kN/m^3 is the unit weight of water; $u_{ni}(r, z, t)$ is the excess pore-water pressure at an arbitrary point in the natural soil zone; $\overline{u_i}$ is the average excess pore-water pressure of soil at a given depth, as given by

$$\overline{u_i} = \frac{1}{\pi(r_e^2-r_w^2)}\left(\int_{r_w}^{r_s} 2\pi r u_{si}\mathrm{d}r+\int_{r_s}^{r_e} 2\pi r u_{ni}\mathrm{d}r\right) \tag{2}$$

where $u_{si}(r, z, t)$ is the excess pore-water pressure at an arbitrary point in the smear zone.

Consolidation in the smear zone can be expressed as

$$\frac{k_{si}}{\gamma_w}\left(\frac{1}{r}\frac{\partial u_{si}}{\partial r}+\frac{\partial^2 u_{si}}{\partial r^2}\right)+\frac{k_{vi}}{\gamma_w}\frac{\partial^2 \overline{u_i}}{\partial z^2}=m_{vi}\left(\frac{\partial \overline{u_i}}{\partial t}-\frac{\mathrm{d}q}{\mathrm{d}t}\right)\quad r_w\le r\le r_s \tag{3}$$

Flow continuity conditions at the boundary of the vertical drain (i.e., drain resistance) can be expressed as

$$\frac{\partial^2 u_{wi}}{\partial z^2} = -\frac{2}{r_w}\frac{k_{si}}{k_w}\left(\frac{\partial u_{si}}{\partial r}\right)\bigg|_{r=r_w} \tag{4}$$

where $u_{wi}(z, t)$ is the excess pore-water pressure within the vertical drain.

The drainage conditions at the vertical boundary of the influence zone is expressed as

$$\left.\frac{\partial u_{ni}}{\partial r}\right|_{r=r_e} = 0 \tag{5}$$

The drainage boundary conditions at the top and bottom surfaces of soil are listed as follows:

$$\begin{cases} u_{w1}|_{z=0} = 0 \\ \left.\overline{u_1}\right|_{z=0} = 0 \\ \left.\dfrac{\partial u_{wN}}{\partial z}\right|_{z=H_N} = 0 \\ \left.\dfrac{\partial \overline{u_N}}{\partial z}\right|_{z=H_N} = 0 \end{cases} \tag{6}$$

where $H_N = \sum_{i=1}^{N} h_i$ is the whole thickness of soil.

The continuity conditions in the radial direction are as follows:

$$\begin{cases} u_{wi} = u_{si}|_{r=r_w} \\ u_{si}|_{r=r_s} = u_{ni}|_{r=r_s} \\ k_{si}\left.\dfrac{\partial u_{si}}{\partial r}\right|_{r=r_s} = k_{hi}\left.\dfrac{\partial u_{ni}}{\partial r}\right|_{r=r_s} \end{cases} \tag{7}$$

The continuity of excess pore-water pressure and the continuity of flow rate at the interfaces between adjoining soil layers can be expressed as: