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Force equilibrium-based finite displacement analyses for reinforced slopes: Formulation and verification

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ABSTRACT

Formulation and verification for a force equilibrium-based finite displacement method (FFDM) using test results of reinforced model slopes subjected to increasing pseudo-static seismic forces are reported. The FFDM requires, in addition to force equilibrium for a sliced potential failure mass, a hyperbolic shear stress–displacement constitutive law for the backfill soils, a hyperbolic pull-out force–displacement constitutive law for the reinforcement, and a displacement compatibility requirement for adjacent soil slices. As a result, the mobilized reinforcement force is an analytical output, rather than an empiricism-based input as required in conventional limit equilibrium analyses. Analytical results from the FFDM also indicated that a brittle failure is associated with the lightly reinforced failure surface; a ductile failure is associated with the heavily reinforced failure surface, regardless of the extensibility of reinforcement investigated in the present study. Good agreements between the measured and the computed slope displacements and reinforcement forces in response to increases in pseudo-static seismic forces suggest that the FFDM can be used as an analytical tool for evaluating displacements of reinforced slopes subjected to pseudo-static seismic loads.

1. Introduction

Displacement analyses of reinforced slope using sophisticated numerical programs often involve time-consuming processes, including personnel training, data preparation, computing, and the inspection of analytical results (Lee and Chang, 2012; Suksiripattanapong et al., 2012; Chen et al., 2013; Leshchinsky and Ling, 2013). On the other hand, force equilibrium-based methods, including limit equilibrium and limit analyses, are widely used tools (e.g., Leshchinsky, 2009; Mohamed et al., 2013; Allen and Bathurst, 2014) for frequently encountered projects that entail limited ground exploration and analytical work budgets. A method with time- and cost-effectiveness is thus of practical importance, in the sense that the method can provide important information on the displacement of slopes with little additional effort, compared to that required in limit equilibrium analyses. To this end, Srbulov (1995) first incorporated stress–strain relationships in a limit equilibrium-based slice method for calculating local safety factors slopes. This method features: (1) a statically determinate force system, (2) satisfying force and moment equilibrium, (3) solving 3ns (ns: total number of slices) simultaneous nonlinear equations, (4) using 'shear–strain (or shear–displacement) ratios to relate local F_s for adjacent segments of slip plane, and (5) using exponential functions to express the stress–strain (or stress–displacement) relationship of slope materials. Shortcomings of Srbulove's method are associated with (3)–(5), i.e., solutions for nonlinear simultaneous equations are initial-guessing-dependent, requiring additional judgments on an acceptable solution among the candidates (e.g., Leshchinsky and Huang, 1992); the 'shear–strain ratio' for adjacent slices was not well-defined and was an empiricism-based input. McCombie (2009) modified Srbulove's method to overcome the above-mentioned shortcomings using the following measures: (1) a wedge-by-wedge equilibrium calculation technique based on force equilibrium, ignoring the moment equilibrium to alleviate the problem of solving 3ns nonlinear simultaneous equations, (2) the introduction of 'compatibility requirements' based on hodographs, in lieu of the empiricism-based input of 'shear strain ratio', and (3) using a linear shear stress–displacement relationship for local safety factor and displacement calculations. Although McCombie's method improved the applicability of that of Srbulove, some aspects still need to be addressed: (1) lack of a concept of incremental slope

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displacement induced by external or internal factor changes; (2) the adequacy of using linear stress–displacement relationships for slope materials. To overcome the above referenced difficulties, Huang (2013a,b) proposed a force equilibrium-based finite displacement method (FFDM) for slope displacement analyses. The FFDM incorporates non-linear shear stress–displacement relationships as constitutive laws for slope materials and a displacement compatibility requirement for the sliding soil blocks. As a result, the sliced potential failure mass is a static determinate system, and local displacements and safety factors along the failure surface can be computed. The proposed FFDM for reinforced slopes has the following features that are distinct from the existing stability analysis methods.

(1) Providing local displacements and safety factors along the failure surface instead of an averaged safety factor for the entire failure surface as is provided by conventional limit equilibrium methods.
(2) Providing slope displacements with little additional time and effort, as compared to conventional slope stability methods. The computer time needed in calculating slope displacements using non-linear stress–displacement relationships and non-linear pull-out force–displacement constitutive laws is no more than that needed in a conventional limit equilibrium calculation for a constant value of safety factor.
(3) Utilizing the shear stress–displacement relationships obtained from large-scale direct shear tests as the constitutive law for the analyzed material. This is similar to that used in the discrete element method (DEM) in which stress–displacement relationships are used to obtain normal and shear spring constants under small displacement conditions. The FFDM uses stress–displacements to determine the shear displacement along a failure surface with relatively large shear displacements.
(4) Adopting the notion of incremental slope displacement or cumulative slope displacement between two different internal and/or external conditions. Huang (2013a) verified this concept based on a monitored case history of a highway slope displacements induced by an intensive rainfall (or a rise in the groundwater table).

The present study is a follow-up study of Huang (2013a,b) with the following new feature.

(5) Utilizing the pull-out force–displacement relationships obtained from pull-out tests for reinforcing material. The mobilized reinforcement force can be computed based on the interaction between the shear displacement of soils and the pull-out displacement of reinforcement. This has not been attempted in previous studies using FEM, limit equilibrium-based methods or limit analyses.

2. Force equilibrium-based finite displacement method (FFDM)

The FFDM is briefly described here. Detailed formulations for the FFDM have been given by Huang (2013a,b). A local stress-based safety factor (FS_i) is defined for the potential failure surface shown in Fig. 1 as:

$$\mathrm{FS}_i = \frac{\tau_{\mathrm{f}_i}}{\tau_i} = \frac{S_{fi}}{S_i} \tag{1}$$

i: slice number ($i = 1, 2, \ldots, \mathrm{ns}$); τ_i: mobilized shear force at the base of slice i ($=S_i/l_i$); and l_i: length of base of slice i.

The ultimate shear stress at the base of slice i is based on Mohr–Coulomb's failure criterion:

$$S_{fi} = \tau_{fi} \cdot l_i = C_i + N'_i \cdot \tan\phi \tag{2}$$

where

$$C_i = c \cdot l_i = c \cdot B_i \cdot \sec\alpha_i \tag{3}$$

where B_i: width of slice i; c: cohesion intercept; ϕ: internal friction angle of backfill; and α_i: inclination angle of the base of slice i.

The effective normal force acting on the base of slice i (N'_i) is expressed as:

$$N'_i = [(1-k_\mathrm{v}) \cdot W_i + T_i \sin\beta_i - S_i \cdot \sin\alpha_i - U_i \cdot \cos\alpha_i] \cdot \sec\alpha_i \tag{4}$$

$$U_i = u_i \cdot l_i = u_i \cdot B_i \cdot \sec\alpha_i \tag{5}$$

where k_h, k_v: horizontal (positive leftward) and vertical (positive upward) seismic coefficients, respectively. W_i: weight of slice i; T_i: mobilized reinforcement force at slice base i (to be discussed later); β_i: dip angle of reinforcement at the base of slice i. The failure shear strength at the base of slice i, namely, S_{fi}, can be expressed as:

$$S_{fi} = \frac{C_i + [(1-k_v) \cdot W_i + T_i \sin\beta_i - U_i \cos\alpha] \cdot \sec\alpha_i \cdot \tan\phi}{1 + \frac{\tan\alpha_i \cdot \tan\phi}{\mathrm{FS}_i}} \tag{6}$$

The differential of horizontal inter-slice forces for slice i, namely, $\Delta E_i (=E_i - E_{i-1})$, is expressed as:

$$\Delta E_i = -\frac{S_{fi}}{\mathrm{FS}_i} \cdot \sec\alpha_i + [(1-k_\mathrm{v}) \cdot W_i] \cdot \tan\alpha_i + T_i \cdot (\sin\beta_i \cdot \tan\alpha_i - \cos\beta_i) + k_\mathrm{h} \cdot W_i \tag{7}$$

A conventional constant value safety factor (F_s) using the simplified Janbu's (1973) method (can be derived based on the summation of ΔE_i, i.e., $\Sigma\Delta E_i = E_\mathrm{ns} - E_0 (i = 1 - \mathrm{ns})$ and replacing FS_i with F_s as:

$$F_\mathrm{s} = \frac{\sum \left\{ \frac{C_i + [(1-k_\mathrm{v}) \cdot W_i + T_i \cdot \sin\beta_i - U_i \cdot \cos\alpha_i] \cdot \sec\alpha_i \cdot \tan\phi}{1 + \frac{\tan\alpha_i \cdot \tan\phi}{F_s}} \right\} \cdot \sec\alpha_i}{E_0 - E_\mathrm{ns} + \sum[(1-k_\mathrm{v}) \cdot W_i \cdot \tan\alpha_i + T_i \cdot (\sin\beta_i \cdot \tan \cdot \alpha_i - \cos \cdot \beta_i) + k_\mathrm{h} \cdot W]} \tag{8}$$

where E_0, E_ns: horizontal boundary force applied at upper and lower ends, respectively, of a potential failure surface (Fig. 1).