Mathematical concepts

Gary Thomas

Richard Isaacs

Abstract

The purpose of mathematics is to apply specific techniques to solve problems. In medicine and the biomedical sciences, the answers to those problems may be solved using logic, algebra, geometry or statistics. The aim of this article is to review some basic mathematical concepts and demonstrate their relevance to current anaesthetic and critical care practice. When one variable y is mathematically related to another variable x, this relationship can be represented by the algebraic equation $y = \text{fn}(x)$. The first variable is said to be a function (fn) of, or dependent on, the second. These relationships can be represented graphically by plotting the dependent variable on the vertical y-axis against the independent variable x on the horizontal axis. The graphic relationships explored in this article include the straight line, rectangular hyperbola, the sigmoid curve, exponentials and the sine wave. The concepts of differentiation, integration and trigonometry are also touched upon.

Keywords exponentials; linear relationships; mathematical concepts; mathematical functions; rectangular hyperbola

The purpose of mathematics is to apply specific numeric techniques to solve problems. In medicine and the biomedical sciences, the answers to those problems may be solved using logic, algebra, geometry or statistics. The aim of this article is to review some basic mathematical concepts and demonstrate their relevance to current anaesthetic and critical care practice.

Mathematical operators

The most basic operators include +, −, * or/. Typical comparison operators include the symbols <, >, = and ≤ (less than or equal to).

More complex operators are used for the differentiation and integration of values.

***Gary Thomas**, MB, BCh (Wales), FRCA is a Consultant Anaesthetist at Bro Morgannwg NHS Trust, Wales. He graduated from the Welsh National School of Medicine in 1981, and trained in anaesthesia in Wales, London and Melbourne. His main interests are anaesthesia for vascular surgery and teaching. He is currently a final FRCA examiner. Conflicts of interest: none declared.*

***Richard Isaacs**, BSc (Hons), MB, BChir (Cantab), FRCA is a Specialist Trainee 4 in Anaesthesia currently on the Welsh rotation. He graduated from the University of St Andrews and Cambridge University Medical School. Conflicts of interest: none declared.*

Mathematical expressions

Expressions are composed of one or more operands (usually numbers) whose values are combined by operators. They are used to assign values to variables.

Functions

When one variable, y, is mathematically related to another variable, x, it can be represented by the algebraic equation:

$$y = \text{fn}(x) \text{ or } y = x$$

In this simple example, the parameter y is a direct function (fn) of x.

Powers (exponents)

These are algebraic functions of the form $y = \text{fn}(x^n)$, where x is the fixed base and n can be any real number as the variable power. They form the basis of the Système Internationale (SI) prefixes used in mathematics and science:

10^{-1}	deci	10^{1}	deca
10^{-2}	centi	10^{2}	hecto
10^{-3}	milli	10^{3}	kilo
10^{-6}	micro	10^{6}	mega
10^{-9}	nano	10^{9}	giga
10^{-12}	pico	10^{12}	tera

Logarithms

Logarithms (logs) are functions that are used to simplify or compress a dataset and to facilitate mathematical calculations. A logarithmic scale is not linear.

Logarithms to the base 10 ($\log_{10}$)

The logarithm of a number (a) with a fixed base (e.g. 10) is the power to which the base must be raised to give that number. Logarithms to the base 10 will therefore convert successive powers of 10 into real numbers. The negative log to the base 10 simplifies the measurement of the hydrogen ion concentration to produce the pH scale, e.g.:

$\log_{10}a = x$,	therefore, $10^x = a$		
$\log_{10}1 = 0$	$10^0 = 1$		
$\log_{10}10 = 1$	$10^1 = 10$	$\log_{10}0.1 = -1$	$10^{-1} = 0.1$
$\log_{10}100 = 2$	$10^2 = 100$	$\log_{10}0.01 = -2$	$10^{-2} = 0.01$
$\log_{10}1000 = 3$	$10^3 = 1000$	$\log_{10}0.001 = -3$	$10^{-3} = 0.001$
$\log_{10}10{,}000 = 4$	$10^4 = 10{,}000$	$\log_{10}0.0001 = -4$	$10^{-4} = 0.0001$

Natural logarithms (ln)

Logarithms to the base ‘e’ are used in differential and integral calculus with exponential functions. Euler’s number (e) is a mathematical constant, which is an irrational number (it cannot be represented by a simple fraction) and can be calculated using the following equation:

$$e = 1 + 1/1! + 1/2! + 1/3! + 1/4! + 1/5! \ldots \text{etc.}$$
$$e = 2.718281$$

Note that '!' means factorial, which is the product of all integers less than and equal to n (i.e. 3! is 1 × 2 × 3).

Pi (π)

Pi is another important mathematical constant. It can be calculated by dividing the circumference of a circle by its diameter ($\pi = 3.14159$). This number has been computed to more than 10^{12} digits with no recognizable pattern of repetition and has been the subject of fascination to mathematicians for centuries. This constant is used in many scientific and mathematical fields including geometry, trigonometry, number theory, physics, probability and statistics.

Mathematical relationships

Linearity

The relationship $y = (\text{fn})x$ can be represented graphically by plotting the dependent variable on the vertical y-axis against the independent variable x on the horizontal x-axis. This association between y and x is a straight line that has an intercept that passes through zero and a gradient (fn). The gradient or slope is also known as the modulus (m).

A plot of the relationship $y = \text{m}(x) + \text{c}$ is also linear, with both m and c (intercept on the y-axis) as constants (Figure 1a).

The slope of a straight line may be positive or negative. Measurement systems depend on linearity of calibration 'curves' to ensure reliable and accurate readings. Inaccuracies may result if there is a gradient or zero offset drift.

Further examples of relevant mathematical relationships are listed in Table 1.

The rectangular hyperbola

A hyperbola is the curve of a conic section consisting of two open branches, each extending to infinity. The simplest form of a rectangular hyperbola is defined through the graph of the function (Figure 1b):

$$y = 1/x$$

The line of the graph approaches the x- and the y-axes but does not intersect them. The axes are called asymptotes. It is sometimes confused with the shape of an exponential curve but is mathematically distinct.

Further properties of a rectangular hyperbola include vertical asymptote at $x = 0$, horizontal asymptote at $y = 0$ and there are no stationary points (where the gradient is zero). Mathematical relationships that yield a hyperbolic shape include:

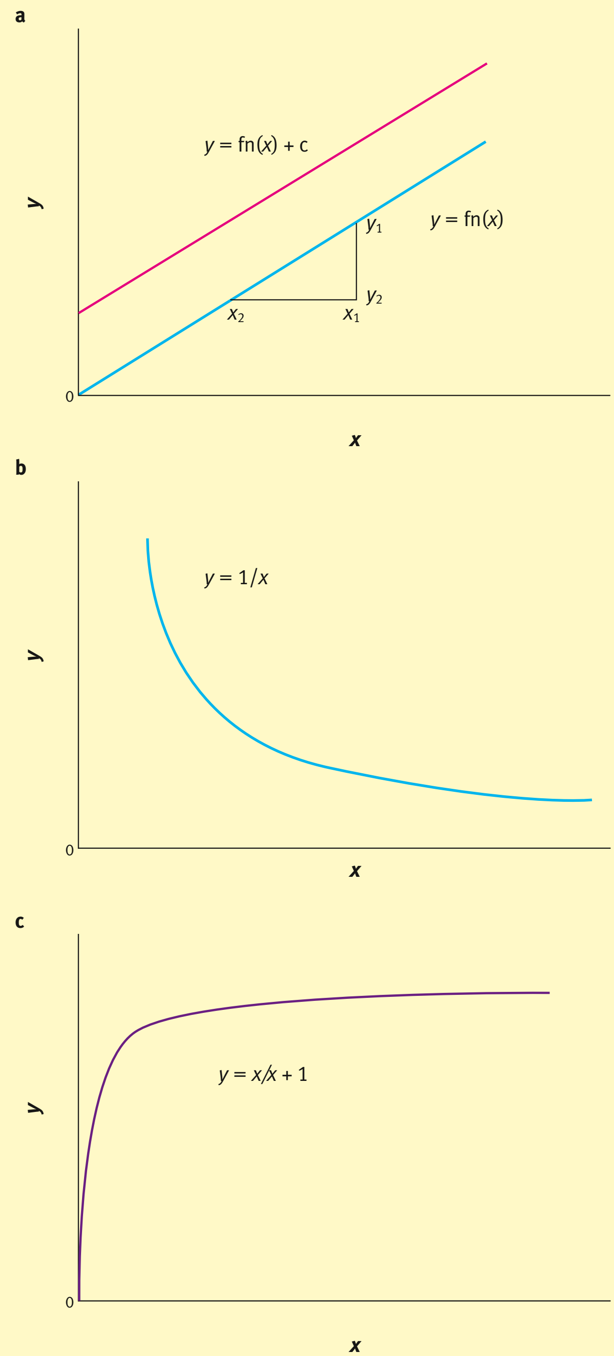
- gas pressure plotted against gas volume (Boyle's law)
- reaction velocity plotted against substrate concentration ('simple' enzyme reaction; Michaelis–Menten kinetics)

$$V_0 = V_{max}\,[S]/K_m + [S]$$

where K_m is the Michaelis–Menten constant, V_{max} is the maximum velocity and [S] is the substrate concentration

- dose of a pure agonist plotted against response (dose–response curve).

Graphical relationships



a The two functions $y = \text{fn}(x)$ and $y = \text{fn}(x) + \text{c}$. The independent variable is x, and y is dependent. The slope fn or the modulus (m) is $y_1 - y_2/x_1 - x_2$. **b** The function $y = 1/x$. **c** The function $y = x/x + 1$

Figure 1

The last two examples are described by the equation (Figure 1c):

$$y = x/1 + x$$

Transformations of a dose–response curve

The shape of a dose–response curve for a pure agonist can be transformed into a sigmoid shape by plotting the x values on