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Geotextiles and Geomembranes 24 (2006) 52–57

Geotextiles and Geomembranes

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Technical Note

# Numerical procedures for deformation calculations in the reinforced soil walls

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Received 28 January 2005; received in revised form 10 July 2005; accepted 16 July 2005
Available online 19 October 2005

**Abstract**

This study presents a membrane analogy method to evaluate the deflection of fabric-reinforced earth walls. The resulting equations were solved using a finite difference scheme to obtain the deflection. The numerical results were compared with a full-scale study. The comparisons show good performance of the model.


*Keywords:* Geosynthetic; Soil wall; Membrane analogy; Numerical analysis; Case study

## 1. Background

Lower cost, lightweight, improved durability, high frictional characteristics, and the relative ease associated with handling and transportation have contributed to the rapid growth of polymeric fabrics in reinforced earth wall technology over the last few decades. However, because of their lower moduli much greater strains are induced in fabric-reinforced earth walls. Therefore, deformations need to be considered to ensure that the deflections at the face of the wall are tolerable.

Field measurements of tensile force distribution have shown it to be nonlinear with the maximum occurring at a distance away from the face of the wall (Floss and Thamm, 1979). Several researchers have accounted for the non-linear tensile stress distribution using a membrane analogy (Love et al., 1987; Bourdeau, 1989; Espinoza, 1994; Shukla and Chandra, 1994, 1995; Yin, 1997a, b). The resulting governing equations have been solved using finite difference as well as finite element numerical schemes. These methods have focused on the applications on the sub-grade deflection.

This study used finite difference scheme of a membrane analogy to calculate the short-range deflections in the fabric and in the face of the reinforced earth wall. The results are compared with a full-scale test results reported by Public Works Research Institute in Japan (PWRI), 1997 and those predicted by Rowe and Skinner (2001) using finite element method.

## 2. Deformation of the fabric based on membrane analogy

The flexibility of the fabric along with the conditions of the load and support conditions leads to the development of a membrane type behavior. This had prompted a number of researchers to apply the membrane analogy to study the deflection of highway subgrades reinforced with fabric (Love et al., 1987; Bourdeau, 1989, Espinoza, 1994; Shukla and Chandra, 1994, 1995; Yin, 1997a, b). The membrane analogy is applied here to the case of wall deflection.

Fig. 1 shows a two-dimensional plane strain of the static equilibrium of an elastic membrane. Using the stochastic stress diffusion theory (Sergeev, 1969; Harr, 1977) for two-dimensional plane strain conditions, the expected vertical stress $\bar{\sigma}_{\mathrm{v}}(x, z)$ at a point (defined by the coordinate $x$ and $z$) given as (Bourdeau, 1989)

$$\frac{\partial \bar{\sigma}_z}{\partial z} = D\frac{\partial^2 \bar{\sigma}_z}{\partial x^2}, \tag{1}$$

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0266-1144/$ - see front matter 
doi:10.1016/j.geotexmem.2005.07.001

where $D$ is the coefficient of diffusion, which governs the rate at which the upper soil layer spreads the applied surface load. It can be assumed to be related to, $K_0$, the coefficient of earth pressure at rest and the depth as (Bourdeau, 1989)

$$D = K_0 z. \tag{2}$$

Using Gaussian distribution (Harr, 1977), the expected vertical stress under an applied pressure, $p$, uniformly distributed over a strip of width $2a$, can be evaluated as

$$\frac{\bar{\sigma}_z}{p} = \psi\left(\frac{x+a}{z\sqrt{K_0}}\right) - \psi\left(\frac{x-a}{z\sqrt{K_0}}\right), \tag{3}$$

where $\psi$ is the cumulative Gaussian distribution function:

$$\psi(x) = \int_0^x \mathrm{e}^{-t^2/2}\,\mathrm{d}t. \tag{4}$$

The compressible soil is assumed to offer a reaction to the loading pressure proportional to its deflection as in Winkler model as

$$\bar{\sigma}_{z,2} = k_s \omega(x), \tag{5}$$

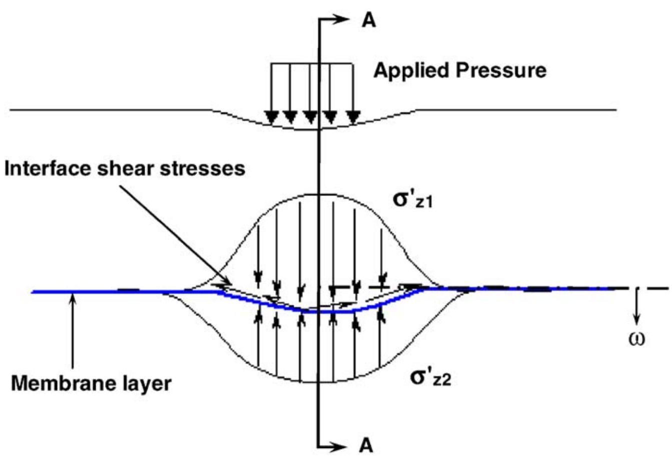


Fig. 1. Two-dimensional plane strain model of static equilibrium of an elastic membrane (modified after Bourdeau, 1989).

where $\bar{\sigma}_{z,2}$ is the vertical stress at the fabric–lower soil layer interface, $\omega(x)$ the membrane deflection, and $k_s$ the coefficient of subgrade reaction.

The interface frictional stress at the sand fabric interface is given by the Mohr Coulomb as:

$$\tau(x) = \mu(\bar{\sigma}_{z,1}(x) + \gamma H_1). \tag{6}$$

where $\bar{\sigma}_z(x)$ is the vertical stress at the backfill underneath fabric interface, $\gamma$ the unit weight of the backfill upper layer, $H_1$ the thickness of the soil column above the fabric, and $\mu$ the interface friction coefficient.

The force acting on the deflected membrane is as shown in Fig. 2. The equilibrium of forces in the horizontal direction results in (Fig. 2):

$$T_\mathrm{H}(x) + \int_0^x \tau_\mathrm{H}(x)\,\mathrm{d}x = T_0, \tag{7}$$

where $\tau_\mathrm{H}(x)$ is the horizontal component of the friction stress at the interface, $T_\mathrm{H}(x)$ the horizontal component of the tensile force in the membrane, and $T_0$ the horizontal tensile force, at the origin of coordinate (i.e., section A–A in Fig. 1).

Equilibrium in vertical direction is written as

$$T_\mathrm{V}(x) - \int_0^x (\bar{\sigma}_{z,1} - \bar{\sigma}_{z,2})\,\mathrm{d}x + \int_0^x \tau_\mathrm{V}(x)\,\mathrm{d}x = T_0, \tag{8}$$

where $\tau_\mathrm{V}$ is the vertical component of the frictional stress.

Taking the derivative of Eq. (8) and using Eq. (5) results in

$$\frac{\mathrm{d}T_\mathrm{V}}{\mathrm{d}x} + \tau_\mathrm{V}(x) + k_s\omega(x) = \bar{\sigma}_{z,1}. \tag{9}$$

From the geometry of forces and deflection (Fig. 2):

$$\frac{T_\mathrm{V}}{T_\mathrm{H}} = \frac{\mathrm{d}\omega}{\mathrm{d}x}. \tag{10}$$

Rewriting Eq. (10) and taking the implicit derivative with respect to $x$:

$$\frac{\mathrm{d}T_\mathrm{V}}{\mathrm{d}x} = \frac{\mathrm{d}(T_\mathrm{H}\mathrm{d}\omega)}{\mathrm{d}x} = \frac{\mathrm{d}T_\mathrm{H}}{\mathrm{d}x}\frac{\mathrm{d}\omega}{\mathrm{d}x} + T_\mathrm{H}\frac{\mathrm{d}^2\omega}{\mathrm{d}x^2}. \tag{11}$$

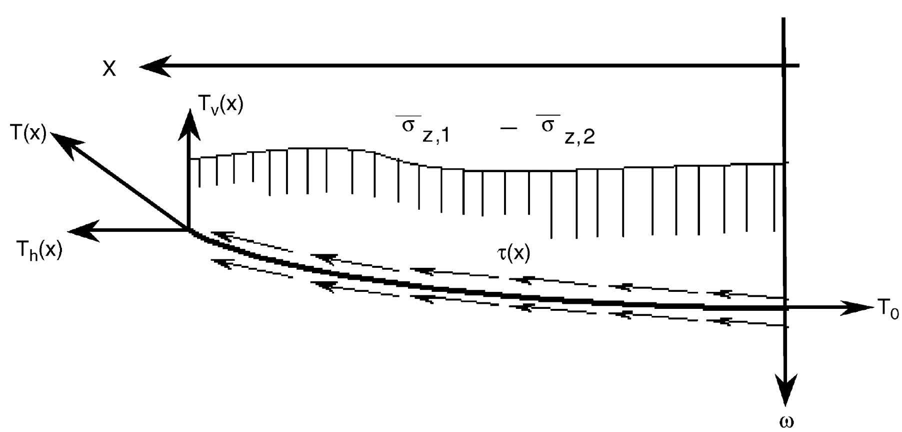


Fig. 2. Forces acting on the deflected membrane (redraw after Bourdeau, 1989).