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Buckling strength of tapered bridge girders under combined shear and bending

Metwally Abu-Hamd [a], **Farah F. El Dib** [b,*]

[a] *Structural Engineering Dept., Cairo University, Egypt*
[b] *Steel Constructions Institute, HBRC, Giza, Egypt*

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Abstract This paper represents the finite element results for the local buckling of tapered plate girders subjected to combine pure bending and shear stresses. An idealized model is developed representing the loading of the tapered panel that generates uniform normal stresses due to flexure, or uniform and constant shear stresses in the case of shear. Eigen-value analysis was performed for several tapered web plate girders that have different geometric parameters. A parametric study is made to reduce the FE model size showing the effect of decreasing the tapered panel adjacent straight panels, maintaining the same result accuracy as a complete girder model. The combined buckling capacity of bending and shear is determined by applying all possible load pattern combinations, together with different interaction ratios. An analysis study is presented to investigate the effect of the tapering angle on the combined bending–shear capacity of the girder. The study also includes the effect of the flange and web slenderness on the local buckling of the girder. Considering residual stresses as part of the loading stresses, the analysis procedure is repeated for some cases, and the effect of combining of the residual stresses together with the external loads is found. Empirical approximate formulae are given to estimate the combined flexure–shear buckling resistance of the tapered girder safely.

Introduction

Plate girders are widely used in steel structures, especially when there is a need to resist high loads, such as in bridges. The use of deep slender web girders is often chosen to give an adequate design. The tapering of the web depth can be implemented to avoid the use of excessive material quantities. Due to the web slenderness, the girder panel usually suffers instability due to the presence of normal and/or shear stresses. Normal stresses are usually induced due to flexural stresses, or due to the inclined component of shear stresses. For relatively short

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[*] Corresponding author.
E-mail addresses: abuhamd@eng.cu.edu.eg (M. Abu-Hamd), farah_fayrouz@yahoo.com (F.F. El Dib).

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panels, flexural stresses may cause compression buckling of the web or local buckling of compression flange. Shear stresses mainly cause shear buckling of the web.

Current design codes such as EN 1993-1-5 [1], AASHTO [2], are based on the theoretical and numerical research of prismatic girders, and they determine bending and/or shear resistance of tapered girders as prismatic ones.

This paper is divided into three parts. The first part demonstrates the finite element model, as well as the idealized loading model that is aimed at generating uniform flexural stresses without shear stresses, or vice versa. Hence, by applying both bending and shear patterns with different loading ratios and various direction combinations, the interaction bending–shear resistance is determined for loading ratios that express the accurate effect of bending and/or shear stresses on the elastic buckling strength of the tapered panel. The second part represents the analysis studies performed using the FEM Eigen-value analysis. Bending–shear interaction diagrams are plotted for different loading ratios, including several web and flange slenderness ratios, panel length ratios and tapering ratio. Furthermore, the obtained results investigate the effect of web tapering angle on the interaction capacity of the tapered plate girder.

Finally discussion and conclusions are given upon the performed analysis, and recommendations are given for future work.

Literature review and state of the art

Eid [3], presented the first known analysis of tapered thin plates using the finite difference method. He established numerical expressions for the inclined plate edges. He also solved the bending of plates under randomly distributed lateral loads, as well as buckling problems of tapered thin plates subjected to in plane acting loads. He compared the tapered thin plates with equivalent rectangular ones having the same critical load under different types of loading. He considered the effect of buckling shape and the number of half waves on the minimum critical buckling stresses.

Mirambell and Zarate [4], Estrada [5], and Chacon [6] presented a series of research papers concerning the elastic and inelastic ultimate strength for shear buckling of tapered web plate girders. Estrada [5] developed an expression to determine the critical shear buckling stress in steel web panels. This expression takes into account the effects of material nonlinearity together with the actual boundary conditions of the web panel.

Mirambell et al. [7–9], introduced an analytical formulation to determine the shear elastic buckling stress factor including the effect of flange slenderness and tapering angle. They also introduced an interaction formula for bending–shear interaction that depends on tension field theory. They considered that the ability to represent the ultimate shear resistance of the tapered girder for the given model depends on the fact that when the maximum shear resistance is reached, the bending moment in the largest cross section is null. Recently, Mirambell and Zarate [8] continued their research on the shear resistance of tapered web plate girders considering geometrical imperfections and residual stresses numerically, and comparing the results to experimental tests.

Abu-Hamd and Abu-Hamd [10] conducted full girder model demonstrating and determining the effect of flange slenderness on the pure flexural or pure shear elastic local buckling of a tapered panel. They were the first to introduce a model with a self-equilibrated loading pattern to achieve pure flexural or shear stresses in order to study each buckling case solely. The loading pattern developed in this paper depends on the same principal of equilibrating the loads such that the stresses are pure, but with the pattern configuration implemented to increase stress uniformity for either shear stresses or flexural normal stresses. They compared the results of pure bending and pure shear to the AASHTO [2] specifications. In addition, they conducted a parametric study and evaluated the buckling stress factors of pure shear and pure bending by varying the tapering ratio and web and/or flange slenderness. They recommended an investigation of the effect of combined shear and bending and post buckling behavior. Herein, the study aims at investigating the effect of combined shear and bending on the stability of tapered web plate girders, including geometrical parameters such as the web and flange slenderness, as well as, the tapering ratios.

Finite element analysis

Elastic buckling strength

The theoretical elastic buckling stress of a rectangular plate, σ_{cr}, is given by the widely known formula [11]:

$$\sigma_{cr} = k_\sigma \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{d}\right)^2 \tag{1}$$

where E is the modulus of elasticity, v is Poisson's ratio, t is the thickness of the plate, d is the width of the plate, and k_σ is the plate buckling factor, which depends on the type of stress distribution and the edge support conditions.

Finite element analysis may be used effectively to obtain the elastic buckling stress under a wider scope of design variables related to the applied stresses and actual boundary conditions Earls [12], Ziemian [13], and Real et al. [14]. The buckling stress is obtained by solving the Eigen-value problem:

$$K_E = \lambda K_G \tag{2}$$

where K_E is the elastic stiffness matrix, K_G is the geometric stiffness matrix, and λ is the Eigen-value, which represents the buckling load factor. The corresponding Eigen-vector represents the mode shapes of the buckled plate.

Idealized loading of the model

The plate buckling solution initiated by Timoshinko [11] is based on homogeneous and pure stresses, normal or shear, in one or two directions, and acting in the middle plane. Later different code provisions adopted simplified interaction analyses between different stress cases to simplify the design approach. Abu-Hamd and Abu-Hamd [10] presented idealized load patterns providing pure normal or shear stresses and calculated accurate basic buckling analysis for either shear or bending.

They found that interaction-buckling analysis is very sensitive and requires accurate models.

The given idealized loading in Abu-Hamd and Abu-Hamd [10] is further improved to achieve acceptable interaction accuracy for the large amount of cases expected. The accuracy of the proposed idealized loading is assured as follows: