HBRC Journal (2014) **10**, 108–116

Housing and Building National Research Center

HBRC Journal

http://ees.elsevier.com/hbrcj

FULL LENGTH ARTICLE

Parametric study of the structural and in-plane buckling analysis of ogee arches

Ghada M. El-Mahdy *

Structures and Metallic Construction Research Institute, Housing and Building National Research Center (HBRC), 87 El-Tahrir St., Dokki, Giza, PC 11511, PO Box 1770, Cairo, Egypt

Received 24 April 2013; revised 16 July 2013; accepted 20 July 2013

KEYWORDS

Arches;
Buckling analysis;
Finite element analysis;
Geometry;
Ogee shaped arch

Abstract As part of a pilot project an ogee arch is being studied as a self-supporting skin skylight for the Housing and Building National Research Center's (HBRC) patio. The ogee arch consists of a pair of two tangential circular arcs making an arch shape. The geometry of the arch depends on several interrelated variables including the angles subtended by the arcs, the ratio of the radii of the two arcs, and the height of the arch. This paper provides curves for designing the geometry of ogee arches. The structural analysis of two-hinged ogee arches under different cases of loading is outlined deriving expressions for the horizontal base thrust and plotting their graphs. A parametric study of the antisymmetric in-plane buckling behavior of ogee arches is presented using a finite element eigenvalue buckling analysis for several cases of loading. The finite element models consist of beam elements and have varying geometrical dimensions representing different shapes of ogee arches. The structural response of the arches is verified through a linear finite element analysis. The results of the buckling analysis are verified through a nonlinear finite element analysis with initial imperfections. It is found that the buckling load is a function of the ratio of the height-to-base radius of the arch and expressions for the lower bound buckling load are derived as a function of this height-to-base radius ratio.

Introduction

An arch is a planar structure that spans a space and supports a load. The significance of the arch is that it provides an esthetically pleasing shape, as well as, theoretically provides a structure which eliminates tensile stresses in spanning a great amount of open space. The forces are mainly resolved into compressive stresses. By using the arch configuration significant spans can be achieved. However, one downside is that an arch pushes outward at the base, and the horizontal reaction force (or *thrust*) needs to be restrained in some way. Arches can be fixed, hinged,

---

* Tel.: +20 2 37617057x1606, mobile: +20 127 645 9099; fax: +20 2 33351564.
E-mail address: ghadaelmahdy@yahoo.com

Peer review under responsibility of Housing and Building National Research Center.

1687-4048 © 2013 Production and hosting by Elsevier B.V. on behalf of Housing and Building National Research Center.
http://dx.doi.org/10.1016/j.hbrcj.2013.07.001

or have 3 hinges, as shown in Fig. 1. Arches can take several shapes consisting of a combination of lines, arcs of circles, and other curves as shown in Fig. 2.

As part of a pilot project on sustainable or green construction at the Housing and Building National Research Center (HBRC) [1], it is proposed to cover the open patio space at the ground floor of HBRC's main building with a self-supporting skin skylight. The HBRC logo consists of an ogee shaped arch with a symbolic sun behind it, hence the new skylight under consideration could take the shape of an ogee arch. The sun symbolizes renewable energy and light, and the arch itself being symbolic of HBRC's leading role in Egypt in sustainable construction. As there is very little data on ogee arches, the subject of this research is the structural and in-plane buckling analysis of two-hinged ogee shaped arches.

Ogee is a curved shape somewhat like an "S" consisting of two arcs that curve in opposite senses, so that the ends are tangential. In architecture, the term ogee is used for a molding with a profile consisting of a lower concave arc flowing into a convex arc. The ogee arch dates back to ancient Persian and Greek architecture [2] and is also found in Gothic style architecture. Ogee is also a mathematical term meaning "inflection point". In fluid mechanics, the term is used for ogee-shaped aerodynamic profiles, a good example of which is the wing of the Concorde aeroplane. As the upper curves of the ogee arch are reversed, it cannot bear a heavy load. However for the purpose of a self-supporting skin skylight that will only be exposed to its own weight and wind loads, the ogee arch is a suitable solution.

Previous literature

An extensive bibliography on the stability of arches prior to 1970 is given by DaDeppo and Schmidt [3]. The *Handbook of Structural Stability* [4] gives an overview of results of stability research of arches in which either the equations or graphs of the quoted literature are reproduced. An extensive state-of-the-art report on elastic and inelastic stability of arches is given by Fukumoto [5]. Singer et al. [6] provide a chapter on experimental research that has been conducted on arches. King and Brown [7] present a comprehensive study for the practical design of steel curved beams and arches.

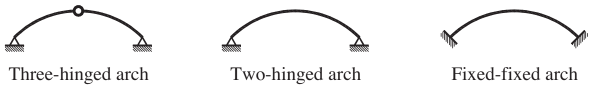

Fig. 1 Statical system of arches.

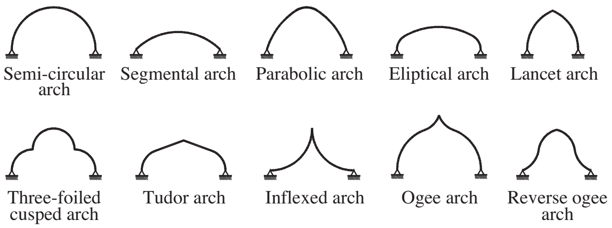

Fig. 2 Common shapes of arches.

Early papers on arch stability devoted to linear stability problems where no bending moments were induced in the arch before buckling were summarized by Austin [8], Austin and Ross [9], and Timoshenko and Gere [10]. More recent results on the stability of tapered arches are reported by Wolde-Tinsaie and Foadian [11]. Nonlinear elastic stability where bending moments are induced in the arch before buckling is handled by Austin and Ross [9]. The problem of unsymmetrical loading was studied by Kuranishi and Lu [12], Chang [13], and Harrison [14]. For the same dead and live load intensities, it was found that unsymmetrically distributed load always governs.

The limit analysis of stocky arches was first presented by Onat and Prager [15]. A more recent theoretical method for calculating the plastic collapse load of stocky arches is given by Spoorenburg et al. [16]. The behavior of slender arches in pure compression is very much like that of a column and it is common to express the buckling strength of such arches in terms of axial thrust at the quarter point of the arch using Euler load [17]. Pi and Trahair [18] and Pi and Bradford [19] studied the in-plane inelastic stability of hinged and fixed circular arches with I-shape cross sections with different load cases and subtended angles. Other nonlinear buckling studies on arches were made by Pi and Trahair [20], Pi et al. [21], and Yau and Yang [22].

International building standards are compared with each other in *Stability of Metal Structures, a World View* [23]. The Eurocode 3, Part 2 [24] provides charts with effective length factors for the elastic in-plane buckling of circular, parabolic, and catenary arches with unmovable supports and several articulations. For tied arches with vertical hangers, effective lengths are also given, as it is a criterion which indicates if the arch is prone to snap-through buckling. AASHTO [25] provides effective-length factors for fixed, two-hinged and three-hinged arches with rise-to-span ratios of 0.1–0.4.

Geometry of ogee arch

The ogee arch is composed of a pair of two discrete circular arcs with independent radii. Hence, there are many geometrical variables to be determined namely, the radius of the lower arc which is half the span of the arch, R_1, the radius of the upper arc, R_2, the angle subtended by the lower arc, 90-α, the angle subtended by the upper arc, β, as well as the overall height of the arch, h. These variables are shown in Fig. 3. From the geometry of the arch the coordinates of the peak of the arch, point 3, can be expressed as

$$x_3 = (R_1 + R_2)\sin\alpha - R_2\sin(\alpha + \beta) = 0 \tag{1}$$

$$y_3 = (R_1 + R_2)\cos\alpha - R_2\cos(\alpha + \beta) = h \tag{2}$$

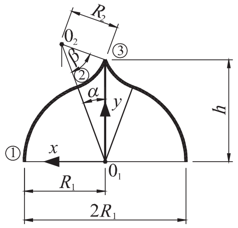

Fig. 3 Geometry of ogee arch.