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FULL LENGTH ARTICLE

New simple equations for effective length factors

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Abstract In most current codes of design steel members and frames, specifications for the design of compression columns or of beam-column use the effective length factor; K. The effective length factor is employed to facilitate the design of framed members by transforming an end-restrained compressive member to an equivalent pinned-ended member. The effective length factor is obtained by solving the exact equations numerically which require many routine calculations or by using a pair of alignment charts for the two cases of braced frames and sway frames. The accuracy of these charts depends on the size of the chart and the reader's sharpness of vision. Instead of using complicated equations or charts, simple equations are required to determine the effective length factor directly as a function of the rotational resistant at column ends (G_A, G_B). In this paper, new simple and accurate equations for effective length factors are presented using multiple regressions for tabulated exact values corresponding to different practical values of the rotational resistance at column ends (G_A, G_B). The investigated equations are more accurate than equations that are recommended in some steel constructions codes. Comparisons between the results of the present equations and those obtained by equations presented in previous researches with those obtained by exact solutions are also given in this paper.

Introduction

The design of a column or of a beam-column starts with the evaluation of the elastic rotational resistance at both ends of the column (G_A, G_B), from which the effective length factor (K) is determined. The mathematically exact equations for braced and sway rigid frames were given by Barakat and chen [6]. These equations require many routine calculations, and it is well suited for tedious column and beam-column calculations. The other way to determine the effective length factor (K) is the using of a pair of alignment charts for braced frames and sway frames, which were originally developed by O.J. Julian and L.S. Lawrence, and presented in detail by T.C. Kavanagh [8]. These charts are the graphic solutions of the mathematically exact equations and these are commonly used in most codes as the manual of American institute of steel construction (LRFD and ASD) [1,2] and the Egyptian code of practice for steel constructions (LRFD and ASD) [4,5]. The accuracy of the alignment charts depends essentially on the size of the chart and on the reader's sharpness of vision. Also, having to read K-factors

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from an alignment chart in the middle of an electronic computation, in spreadsheet for instance prevents full automation and can be a source of errors.

Obviously, it would be convenient to have simple equations take the place of the charts which are commonly used in most codes of steel constructions. The American Institute does publish equations but their lack of accuracy may be why they seem not to be used in steel design. Better equations have been available in the French design rule for steel structures since 1966, and have been included in the European recommendations of 1978 [3] are presented by Pierre Dumonteil [7].

In this paper, more accurate closed form equations for the determination of the effective length factors as a function of the rotational resistance at column ends are presented which are simple enough to be easily programed within the confines of spreadsheet cell. For this reason, they may be useful to design engineers.

Background to exact and approximate equations

Consider a column AB elastically restrained at both ends. The rotational restraint at one end, A for instance, is presented by restraint factor G_A, expressing the relative stiffness of all the columns connected at A to that of all the beams framing into A:

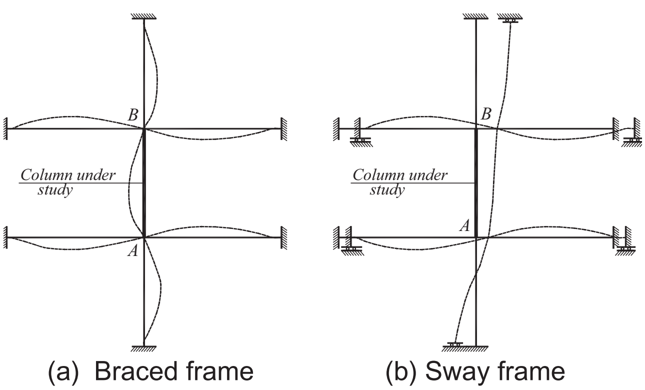

Fig. 1 Braced and sway frames.

$$G_A = \frac{\sum(I_C/L_C)}{\sum(I_b/L_b)} \quad (1)$$

In the European Recommendation, another two factors β_A and β_B are used (rather than G_A and G_B as in French Rules). The definition of β differs from that of G, since, at each column end:

$$\beta = \frac{\sum(I_b/L_b)}{\sum(I_b/L_b) + \sum(I_c/L_c)} \quad (2)$$

The mathematical relation between G and β is simple:

$$\beta = 1/(1+G) \quad (3)$$

Europeans tend to prefer β to G because a hinge means $\beta = 0$ and fixity means $\beta = 1$. Obviously, the K-factor will be the same if the same elements are introduced in G and β.

Braced frames

Braced frames are frames in which the side sway is effectively prevented as shown in (Fig. 1a), and, therefore, the K-factor is never greater than 1.0. The side sway prevented alignment chart is the graphic solution of the following mathematical equation:

$$\frac{G_A G_B}{4}(\pi/K)^2 + \left(\frac{G_A+G_B}{2}\right)\left(1-\frac{\pi/K}{\tan(\pi/K)}\right) + 2\frac{\tan(\pi/2K)}{\pi/K} = 1 \quad (4)$$

This equation is mathematically exact, in that certain physical assumptions are exactly translated into mathematical terms. Whether these assumptions can be reasonably extended to a specific structure is a matter for the designer to decide.

For the transcendental Eq. (4), which can only be solved by numerical methods, the French Rules propose the following approximate solution:

$$K = \frac{3\,G_A G_B + 1.4\,(G_A+G_B) + 0.64}{3\,G_A G_B + 2.0\,(G_A+G_B) + 1.28} \quad (5)$$

Sway frames

If a rigid frame depends solely on frame action to resist lateral forces, its side sway is permitted as shown in (Fig. 1b). In this

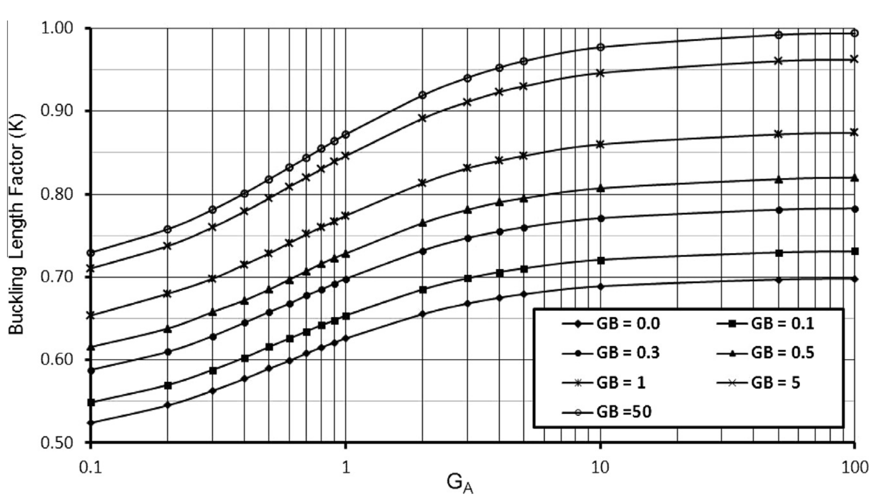

Fig. 2 Buckling length factor (K) for prevent of sway (Braced) columns.