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Kinematics of a Class of Three-Legged Parallel Manipulators by Means of Screw Theory

Cinemática de una clase de manipuladores paralelos compuestos de tres extremidades por medio de la teoría de tornillos

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Abstract

This work is devoted to the velocity and acceleration analyses of a class of three-legged parallel manipulators. The input-output equations of velocity and acceleration are systematically obtained by resorting to reciprocal-screw theory. With the purpose to exemplify the application of the method, a case study is included. The example consists of solving the kinematics of a 3-CRS (Cylindrical + Revolute + Spherical) parallel manipulator. Furthermore, the numerical results obtained via screw theory are verified with the aid of commercially available software.

Keywords:

- parallel manipulator
- klein form
- screw theory
- kinematics

Resumen

Este trabajo se dedica a los análisis de velocidad y aceleración de una clase de manipuladores paralelos compuestos de tres extremidades. Las ecuaciones entrada-salida de velocidad y aceleración se obtienen sistemáticamente recurriendo a la teoría de tornillos recíprocos. Con el propósito de ejemplificar la aplicación del método, se incluye un caso de estudio. El ejemplo consiste en resolver la cinemática de un manipulador paralelo tipo 3-CRS (Cilíndrico + Revoluta + Esférico). Más aún, los resultados numéricos obtenidos vía teoría de tornillos se verifican con la ayuda de software comercial.

Descriptores:

- *manipulador paralelo*
- *forma de Klein*
- *teoría de tornillos*
- *cinemática*

Introduction

Manipulators may be classified as serial, parallel or a combination of both, named hybrid mechanisms. A typical serial manipulator consists of an end-effector connected to the base link by means of a single kinematic chain, in which all the kinematic pairs play the role of active or motive joints. On the other hand, a typical parallel manipulator consists of an end-effector, namely the moving platform, connected to the base link, namely the fixed platform, by means of at least two kinematic chains or limbs. The presence of passive kinematic joints is a characteristic of parallel manipulators.

Without doubt, the most popular type of parallel manipulator is the so called Gough-Stewart platform. This non-redundant in-parallel manipulator consists of a moving platform and a fixed platform connected to each other by means of six extendible limbs or rods, where each limb is actuated independently providing six degrees of freedom to the moving platform. The parallel kinematic device is attributed to Gough (Gough 1957; Gough and Whitehall, 1962) and, incorrectly, to Stewart (1965). These seminal contributions, a universal tire testing machine and a fly simulator, respectively, date back to the 1950s. Despite the indisputable and appreciable benefits of a Gough-Stewart platform such as rigidity and accuracy, one of its main drawbacks, due to the coupled kinematics over the moving platform, is the hazardous task for computing the forward position analysis. In fact, as it is reported by Raghavan (1993), the moving platform can reach up to forty locations when a set of generalized coordinates is given. The problem has been exhaustively addressed, see for instance (Innocenti and Parenti, 1990; Wen and Liang, 1994; Innocenti 1995; Husty 1996; Innocenti 1998), providing excellent partial solutions. However, still in our days a closed-form solution for the forward position analysis seems to be an unrealistic task. Furthermore, limited workspace and a recurrent problem of singular configurations are the main drawbacks of most parallel manipulators. Recently, several robots for industrial purposes have been constructed based on the Gough-Stewart topology: Octahedral Hexapod HOH-600 (Ingersoll), HEXAPODE CMW 300 (CMW), Cosmo Center PM-600 (Okuma), F-200i (FANUC) and so far. These products exhibit an excellent performance, however one cannot ignore that this kind of parallel kinematic structures have a limited and complex-shaped workspace. Furthermore, their rotation and position capabilities are highly coupled and therefore the control and calibration of it are rather complicated demanding the implementation of sensors.

An option to overcome the limitations of Gough-Stewart platforms is the development of parallel manipulators with fewer than six limbs, preserving of course the six degrees of freedom. In this work the velocity and acceleration analyses, of a class of three-legged parallel manipulators are investigated by means of the theory of screws. The study is available for a wide range of parallel manipulators.

Description of the class of parallel manipulators under study

It is known that the pose, position and orientation, of rigid body may be determined by knowing the position of three non-collinear points embedded to it (Merlet, 2004; Gallardo, 2014). Furthermore, it is straightforward to demonstrate that a minimum number of limbs may avoid the possibility of physical interference between the limbs, increasing the workspace of parallel manipulators. Thus the parallel manipulators considered in this study have the following features:

- The moving platform is connected to the fixed platform by means of three limbs.
- In order to simplify the mechanical assembly, avoiding additional conditions of manufacture, the moving platform is connected to the limbs by means of three spherical joints shaping an equilateral triangle.
- The limbs are connected to the fixed platform by means of revolute, cylindrical or prismatic joints.
- Overall, the parallel manipulators considered in the study are of the class 3-XYS, where $\{X, Y\} \in \{R, C, P\}$ while S is a spherical joint where C, R, P and S stand for Cylindrical, Revolute, Prismatic and Spherical joint, respectively.

Some parallel manipulators meeting the characteristics above listed are the following: 3-RRS, 3-RPS, 3-CRS, 3-CPS, 3-RRRPS and 3-RRPS. Note that limited a non-redundant parallel manipulators are considered in the contribution.

Preliminary screw theory

As a consideration for readers unfamiliar with the theory of screws, this section is devoted to review some elementary concepts of it.

In Plücker coordinates a screw, notated as \$, is a six-dimensional vector given by $\$ = (s, s_O)$, where s is named the primal of the screw, $P(\$) = s$, and denotes the direction of the line. Usually the primal part is a unit