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Numerical Infinite Series Solution of the Ground-Return Pollaczek Integral

Solución numérica en series infinitas para la integral de retorno por tierra de Pollaczek

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Abstract

In this paper, the Wedepohl-Wilcox series, proposed for calculating ground-return impedances of buried cables and electromagnetic transients, are analyzed in detail. The origin of this series goes back to the original integral derived by Pollaczek. To enhance the analysis developed here, a numerical comparison between the series, the direct numerical integration of Pollaczek integral, and a proposed hybrid numerical algorithm is presented in this paper. The latter consists on: a) the use of a vector-type efficient algorithm for the converging series for low frequencies, and b) trapezoidal numerical integration for the high frequency range. In addition, and based on the analysis, a criterion for switching between series and direct numerical integration is proposed here.

Keywords:

- cables
- frequency response
- power system transients
- earth-impedance
- ground-return models
- skin-effect

Resumen

En este artículo se analiza con detalle la serie de Wedepohl-Wilcox, propuesta para calcular impedancias de retorno por tierra de cables subterráneos, así como transitorios electromagnéticos. El origen de esta serie se remonta a la derivación original de la integral de Pollaczek. Para mejorar el análisis desarrollado aquí se presenta una comparación numérica entre la serie, la integración directa de la integral de Pollaczek y se presenta un algoritmo híbrido numérico. Este último consiste en: a) el uso eficiente de un algoritmo vectorizado para series convergentes en el rango de baja frecuencia y b) la integración numérica trapezoidal para el rango de alta frecuencia. Adicionalmente, basándose en este análisis, se propone un criterio para switchear entre la solución de la serie y la integración numérica directa.

Descriptores:

- *cables*
- *respuesta en frecuencia*
- *transitorios en sistemas de potencia*
- *impedancia de tierra*
- *modelos de retorno por tierra*
- *efecto-skin*

Introduction

One of the most important techniques, over 85 years old, to calculate the influence of the ground-return on aerial and buried electrical conductors was posted by Von F. Pollaczek in June 1926. In this work, Pollaczek presented a set of integral expressions to evaluate the electric field due to an infinite thin filament of current in the presence of an imperfect conducting ground.

Unless, Pollaczek integrals are accurate enough for many power applications, several authors have developed approximate methods and closed-form solutions to avoid facing these rapidly increasing oscillating integrals.

One important publication related to this topic was published in 1973 by Wedepohl and Wilcox, in this publication, a complete mathematical model based on the modified Fourier integral for the synthesis of travelling wave phenomena in underground transmission systems was proposed. An important contribution in Wedepohl and Wilcox (1973) is the solution of Pollaczek's integral through a set of low frequency infinite series. To the best author knowledge, an efficient solution of the series has not been implemented nor included in any commercial software. Besides, it is argued that the series solution is rather complicated and it is better that the impedance is obtained directly from solving the Pollaczek's integral, numerically.

As a first objective, and inspired on the research in Wedepohl and Wilcox (1973), an efficient numerical implementation of the Wedepohl-Wilcox series solution is developed in this paper for calculating ground-return impedances for underground cables, which can guarantee absolute convergence (Kaplan, 1981).

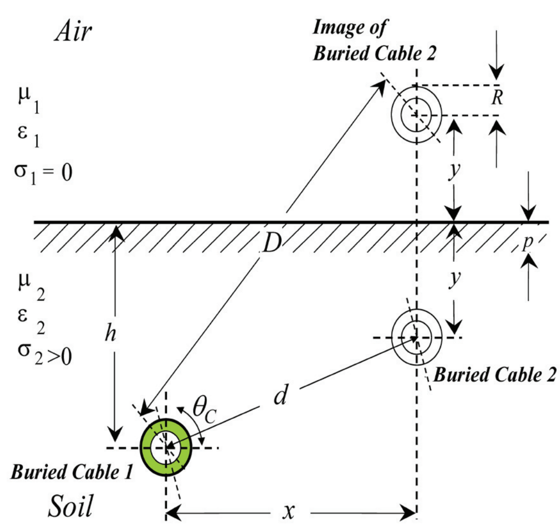

Figure 1. Geometry of the underground system and the image of one of the conductors in the air

As a second objective, a comparison with four different algorithms for solving Pollaczek integral is presented for calculating electromagnetic transients. The first one corresponds to the originally proposed in Wede pohl and Wilcox (1973), i.e., solving the series for low frequencies and using a closed-form solution for the high frequency range. The second algorithm is proposed here and corresponds to a hybrid one. This is based on the rapidly converging series for low frequencies, combined with trapezoidal integration of the unexpanded integral expression for high frequencies (Wedepohl and Wilcox, 1973). The third and the fourth algorithms consist on trapezoidal numerical integration and Gauss-Kronrod routine, respectively, applied directly to the unexpanded and Pollaczek integral, without using approximating series.

As a third objective, the proposed hybrid algorithm is tested for a wide range of practical application cases on transient analysis. This is achieved by using normalized dimensionless variables according to an interpretation for underground cables of the application limits reported in (Ametani *et al*., 2009).

The computational analysis of the studied algorithms is presented here regarding accuracy and CPU-time.

Earth-return impedances

Basic relations

The self and mutual earth-return impedance for a quasi-TEM$_z$ (transversal electromagnetic with respect to "*z*" axis) mode is described by (Figure 1 for reference directions) Wedepohl and Wilcox (1973):

$$Z_g(\omega) = \frac{j\omega\mu}{2\pi}\int_{-\infty}^{\infty}\left[\frac{e^{-|y+h|\sqrt{\alpha^2+1/p^2}}}{|\alpha|+\sqrt{\alpha^2+1/p^2}} + \frac{e^{-|y-h|\sqrt{\alpha^2+1/p^2}} - e^{-|y+h|\sqrt{\alpha^2+1/p^2}}}{2\sqrt{\alpha^2+1/p^2}}\right]e^{j\alpha x}d\alpha \tag{1a}$$

where α is the dummy variable, w represents the angular frequency (in rad/s), μ corresponds to the magnetic permeability (H/m) of the soil, and the complex depth or Skin Effect Layer Thickness (considering displacement currents) is given by many authors (Pollaczek, 1926; Wedepohl and Wilcox, 1973; Kaplan, 1981; Ametani *et al*., 2009; Carson, 1926; Uribe *et al*., 2004; 2000; Dommel, 1986):

$$p = 1/\sqrt{j\omega(\sigma + j\omega\varepsilon_o\varepsilon_r)\mu} \tag{1b}$$