International Journal of Mining Science and Technology 25 (2015) 581–586

Contents lists available at ScienceDirect

International Journal of Mining Science and Technology

journal homepage: www.elsevier.com/locate/ijmst

Analytical solution for stress and deformation of the mining floor based on integral transform

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ARTICLE INFO

Article history:
Received 3 December 2014
Received in revised form 11 January 2015
Accepted 15 February 2015
Available online 6 June 2015

Keywords:
Integral transform
Mining floor
Plastic failure depth
Floor heave
Analytical solution

ABSTRACT

Following exploitation of a coal seam, the final stress field is the sum of in situ stress field and an excavation stress field. Based on this feature, we firstly established a mechanics analytical model of the mining floor strata. Then the study applied Fourier integral transform to solve a biharmonic equation, obtaining the analytical solution of the stress and displacement of the mining floor. Additionally, this investigation used the Mohr–Coulomb yield criterion to determine the plastic failure depth of the floor strata. The calculation process showed that the plastic failure depth of the floor and floor heave are related to the mining width, burial depth and physical–mechanical properties. The results from an example show that the curve of the plastic failure depth of the mining floor is characterized by a funnel shape and the maximum failure depth generates in the middle of mining floor; and that the maximum and minimum principal stresses change distinctly in the shallow layer and tend to a fixed value with an increase in depth. Based on the displacement results, the maximum floor heave appears in the middle of the stope and its value is 0.107 m. This will provide a basis for floor control. Lastly, we have verified the analytical results using FLAC^{3D} to simulate floor excavation and find that there is some deviation between the two results, but their overall tendency is consistent which illustrates that the analysis method can well solve the stress and displacement of the floor.

1. Introduction

After coal resources have been exploited to form goaf, the equilibrium state of the in situ stress field is destroyed, which results in the stress in the coal floor being redistributed. The stress redistribution may make the floor strata undergo large deformation and lead to its destruction. With increasing mining depth, it is important to understand the distribution of the floor stress field, including mastering floor deformation features, determining the floor failure range, predicting floor water invasion, selecting floor location and maintaining the integrity of the floor. Until recently, the most common investigative methods for the analysis of floor strata stress have been numerical simulation and laboratory simulation in physics [1–5]. Only a few analytical solutions have been achieved [6–9]. According to force analysis on a semi-infinite plane, Zhang used elastic–plastic theory to solve the stress field and the maximum failure depth of the floor strata under exploitation conditions [10]. In his research, the abutment pressure in both sides of an arch corner is considered, but the calculation process is very complex for practical application. Zhu employed the elastic theory to analyze stress distribution of the comparative fixed position with the advance of a working face [11]. Based on the distribution of abutment pressure in the mining area, Shi deduced the calculation formula of the non-elastic area [12]. The two researchers both used the abutment pressure to derive the stress field in the floor below the coal seam located in front of the working face. Foreign researchers observed that the stress change was caused as a result of the influence of burial depth and the physical–mechanical properties of overlying strata after the exploitation of the coal seam. In order to derive the stress field of a rectangle roadway, we have simplified the computational model to be a rectangular hole problem in a plane and used the complex function method to obtain the relationship between the stress around roadway and two factors, namely the aspect ratio and lateral pressure coefficient [13–15]. However, this method needs to determine the accurate mapping function of a rectangular hole. The mapping functions of rectangular holes are different with various aspect ratios, so the application of this method is prohibitively difficult. According to field conditions, the mining width is tens of meters and the

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http://dx.doi.org/10.1016/j.ijmst.2015.05.010

height is only 2–3 m. As for the size relationship, we should emphatically consider the effect of mining width.

In brief, the balance state of in situ stress is destroyed by mining the coal seam, which leads to stress redistribution. The weight of the overlying rock passes through the lateral coal wall, which forms the abutment pressure. Former researches assumed the abutment pressure to be linear, and considered that the abutment pressure acted on the floor strata, which caused the floor below the mining area to move upward and form floor heave.

We have considered that floor deformation is caused by excavation unloading. The final stress field in the surrounding rock is a superposition of an in situ stress field and an excavation stress field. We have established a calculation model, as shown in Fig. 1, and used the integral transform method to analytically evaluate the stress field and displacement field of the floor after coal seam mining. By solving an engineering example, the Mohr–Coulomb criterion was employed to derive the plastic failure depth of the floor strata and the floor heave value was also solved, which can provide a theoretical basis for the prevention and control of floor water inversion.

2. Analysis model and approach

2.1. Analysis model

On the basis of the feature of coal excavation, by assuming that the mining width was $2l$, the vertical load was p and the horizontal load was q, the excavation model and the coordinate system were established, as shown in Fig. 1a. The stress state in Fig. 1a is the final stress field after excavation. Fig. 1b expresses the initial stress state. Fig. 1c shows the excavation stress field. The stress field in Fig. 1a can be regard as a superposition of the stress fields in Fig. 1b and c. Because there is no deformation in Fig. 1b, the displacement field can be solved only by the model in Fig. 1c.

Based on the calculation model in Fig. 1c, we can obtain the stress and displacement of the floor strata by solving bi-harmonic Eq. (1) [16] .

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)^2\phi=0 \tag{1}$$

The relationship of stress components and stress function can be expressed as

$$\begin{cases}\sigma_x=\dfrac{\partial^2\phi}{\partial z^2}\\ \sigma_z=\dfrac{\partial^2\phi}{\partial x^2}\\ \tau_{xz}=-\dfrac{\partial^2\phi}{\partial x\partial z}\end{cases} \tag{2}$$

Strain components can be derived by physical equations as follows:

$$\begin{cases}\varepsilon_x=\dfrac{\partial u}{\partial x}=\dfrac{1}{2G}[(1-\mu)\sigma_x-\mu\sigma_z]\\ \varepsilon_z=\dfrac{\partial w}{\partial z}=\dfrac{1}{2G}[(1-\mu)\sigma_z-\mu\sigma_x]\\ \gamma_{xz}=\dfrac{\partial u}{\partial z}+\dfrac{\partial w}{\partial x}=\dfrac{\tau_{xz}}{G}\end{cases} \tag{3}$$

where G is the shear modulus and can be expressed as $G = E/2(1 + \mu)$. μ is Poisson's ratio.

The boundary conditions of this problem are denoted as:

$$\begin{cases}z=0, & \tau_{xz}=0\\ z=0,\quad |x|\leqslant l, & \sigma_z=-p\\ z=0,\quad |x|>l, & w=0\end{cases} \tag{4}$$

2.2. Analysis approach

According to the calculation model, we find that the problem concerns z axis symmetry. Fourier integral transform [17,18] is used to deduce this problem. Fourier cosine integral transform of the function $f_1(x)$ and Fourier sine integral transform of the function $f_2(x)$ can be respectively expressed as:

$$\begin{cases}\bar f_1(\varsigma)=\int_0^\infty f_1(x)\cos\varsigma x\mathrm{d}x\\ \bar f_2(\varsigma)=\int_0^\infty f_2(x)\sin\varsigma x\mathrm{d}x\end{cases} \tag{5}$$

The inversions of Fourier cosine integral transform and Fourier sine integral transform are respectively shown as:

$$\begin{cases}f_1(x)=\frac{2}{\pi}\int_0^\infty \bar f_1(\varsigma)\cos\varsigma x\mathrm{d}\varsigma\\ f_2(x)=\frac{2}{\pi}\int_0^\infty \bar f_2(\varsigma)\sin\varsigma x\mathrm{d}\varsigma\end{cases} \tag{6}$$

3. Problem solving

3.1. Solution for the stress field

Based on Eq. (5), the bi-harmonic Eq. (1) can be changed by Fourier cosine integral transform method as follows:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}-\varsigma^2\right)^2\bar\phi=0 \tag{7}$$

Using the condition that the influence of excavation unloading is limited, the solution to Eq. (7) can be derived as:

$$\bar\phi=(A+Bz)\mathrm{e}^{-\varsigma z} \tag{8}$$

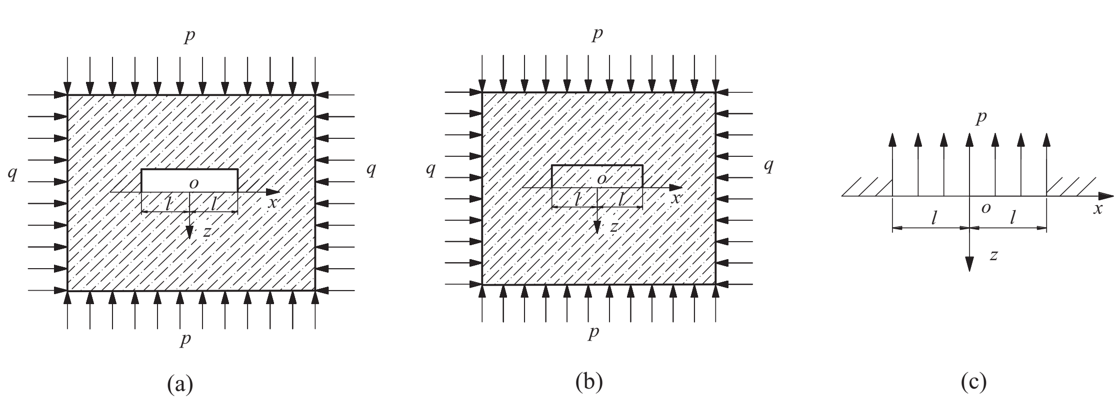

Fig. 1. Calculation model.