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Optimal control of natural resources in mining industry

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ABSTRACT

The paper focuses on the optimal control of natural resources in mining industry. The purpose is to propose an optimal extraction series of these resources during the lifetime of the Mine's maintenance. Following the proposed optimal control model, a sensitivity analysis has been performed that includes the interest rate impact on the optimal solution. This study shows that the increasing of the interest rate stimulates faster extraction of the resources. The discounting factor induces that the resource has to be extracted faster but this effect is counterbalanced by the diminishing returns of the annual cash flow. At higher parameters of "alpha" close to one of the power function about 80% from the whole resource will be extracted during the first 4 years of object/mine maintenance. An existence of unique positive root with respect to return of investment has been proposed and proved by two ways: by the "method of chords" and by using specialized software.

1. Introduction

The optimization today is widely used in many research areas [1–3]. For example, in economics [4], the history shows that the dynamic economics analysis dates from more than 50 years. Ramsey analysed the consumption-saving decision; Hotelling showed how one resource under depletion can be controlled optimally [5,6]. Allen included a chapter on the calculus of variations in his textbook on mathematical economics, which was used by a generation of graduate students [7]. After the 2nd World War, the dynamic programming and optimal control theory tools, which were developed by applied mathematicians, became available to economists [8].

In its earliest development, the dynamic optimization has applied variances calculus which appeared in the 18th and 19th centuries [9]. In contrast to standard calculus, where the function value depends on value of the independent variable, in the variance calculus the "function" value depends on the form of another function. The calculus of variations is much harder than standard calculus theory since it is more difficult to find the optimal form of an entire function than the optimal value of a variable [10,11]. In Ramsey's consumption-saving model, the consumer's utility function depends on the time path of consumption. The goal of dynamic optimization is to find the time path of consumption that maximizes the consumer's lifetime utility. In Hotelling's model of exhaustible resources, dynamic optimization yields the time path of resource extraction that maximizes the value of a resource project, for example a mine [12]. It is interesting to point out that in economics and finance, an optimal path is sought that represents the decision variable as a function of time. As well as the calculus of variations is used in engineering and physics, where the optimal path may have a physical dimension.

In recent years, Stavins used a dynamic analysis of maintenance of natural resources with main focus on the concern of ecology [13]. Lozada applied to the Hotelling's model a new fundamental equation for the time intensity of the function change of optimal values of optimal controls problems [14]. Lyon discussed the role of "costate variable" ("shadow price") for exhaustible and non-exhaustible resources [15]. Piazza & Rappaport considered the optimal extraction problem of resources that cannot be maintained continuously [16]. Some researchers considered resource control in spite of natural facts of given regions. Halkos & Papageorgiou considered the "oil" as non-renewable resource which can be exhausted [17].

One can conclude that the optimal control problem of natural resources is very actual today. This paper shows how the principles of dynamic economics are applied to the solving of natural resources control problems and what the impact of the interest rate is over the optimal control series and the utility function.

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2. Theoretical background

Suppose an optimal control problem in discrete time with the following periods: 0, 1, 2, ..., T, and also assume the following [18]:

$$\Theta = \{0, 1, 2, \ldots, T-1\}$$
$$x_t - \text{n-component vector-column of state variable}; \quad t = 0, 1, 2, \ldots, T \tag{1}$$

u_t-m-component vector-column of control variable; $t = 0, 1, 2, \ldots, T-1$
b_t-s-component vector-column of constants; $t = 0, 1, 2, \ldots, T-1$

In Eq. (1), it is assumed that the state variable x_t is measured at the beginning of each period t and the control u_t is applied during this period t. This notation is shown in Fig. 1:Let's also define continuously differentiable functions:

$$f : E^n \times E^m \times \Theta \to E^n,\ F : E^n \times E^m \times \Theta \to E^1,\ g : E^m \times \Theta \to E^s,$$
$$S : E^m \times \Theta \cup \{T\} \to E^1$$

Hence, the optimal control problem in discrete form can be defined in the following way:

$$\max \left\{ J = \sum_{t=0}^{T} F(x_t, u_t, t) + S(x_T, T) \right\} \tag{2}$$

Subject to constraints:

$$\begin{aligned} &\Delta x_t = x_{t+1} - x_t = f(x_t, u_t, t),\ t = 0, 1, \ldots, T-1 \\ &x_0 - \text{given} \\ &g(u_t, t) \geqslant b_t,\ t = 0, 1, \ldots, T-1 \end{aligned} \tag{3}$$

In our case we consider the problem in a discrete time. For many economics tasks it is necessary to be considered in discrete time since sometimes it is impossible to have records in continuous time [19]. In case of annual information, the time series of resource extraction can be expressed as:

$$\vec{u} = \{u_0, u_1, u_2, \ldots, u_T\} \tag{4}$$

The resource value is the sum of discounted cash flow obtained during the lifetime of the considered object from 0 to T. Suppose in our case $T = 10$ years (i.e. the considered object lifetime is 10 years). Therefore the resource value is [20]:

$$V(x_0, \vec{u}) = \sum_{t=0}^{T} \beta^t C(u_t) \tag{5}$$

where $C(u_t)$-cash flow for $\boldsymbol{t}$-th year; $\beta = 1/(1+r)$-discounting factor; r-interest rate of the resource.

The fundamental concept from the interest rate theory is the net present value in Eq. (5) of the cash flow over time. It should be noted that the arbitrage absence supposes that the value of obligation (agreement, contract) should be the net present value of the cash flow.

Consider a cash flow, i.e. the series of periodic payments $C(u_t)$, discrete in time $t = 0, 1, 2, \ldots, T$. Let the interest rate r is given in discrete complexity and it is applied on the payment periods. Then the net present value is defined by the following expression:

$$V(x_0, \vec{u}) = \sum_{t=0}^{T} \frac{C(u_t)}{(1+r)^t} \tag{6}$$

Another important point related to the cash flow analysis is the return rate. Suppose the following values $C(u_t) > 0,\ t = 0, 1 \ldots T$, and then the return rate $\bar{r}$ can be found by solving of the following non-linear equation:

$$\sum_{t=0}^{T} \frac{C(u_t)}{(1+\bar{r})^t} = 0 \tag{7}$$

and performing a substitution:

$$h = \frac{1}{(1+\bar{r})} \tag{8}$$

then we get an Eq. (7) in the following modified form:

$$\sum_{t=0}^{T} C(u_t) h^t = 0 \tag{9}$$

and this Eq. (9) has unique positive root.

Therefore it is very important to obtain a precise evaluation of the unique positive root of given algebraic equation. In order to evaluate the unique positive root in Eq. (9), then the Eq. (6) has to be expressed in the following way:

$$V(x_0, \vec{u}) - \sum_{t=0}^{T} \frac{C(u_t)}{(1+r)^t} = 0 \tag{10}$$

and it possess unique positive root, where:

$V(x_0, \vec{u})$ – the present value of the project;
$\sum_{t=0}^{T} \frac{C(u_t)}{(1+r)^t}$ – the sum of discounted cash flows over time.

From financial modelling point of view, the cash flow can be expressed as a "power function", i.e. $C(u_t) = u_t^{\alpha}$, which has diminishing returns since $0 < \alpha < 1$. Substituting $C(u_t) = u_t^{\alpha}$ in Eq. (5), then we get the following objective function [8]:

$$V(x_0, \vec{u}) = \sum_{t=0}^{T} \beta^t u_t^{\alpha} \tag{11}$$

The operator can choose the time series of extraction Eq. (4) which maximizes the present value functional: $V(x_0, \vec{u})$ with the following constraint:

$$x_{t+1} - x_t = -u_t \tag{12}$$

The term u_t is a control variable in our study. It should be noted that Eq. (12) is not a true constraint that limits the rate of change of the resource reserve through some technological relationship. Eq. (12) says that the decision-maker controls the reduction in the resource reserve from one year to the next. As well as, a numerical solution to a dynamic optimization problem requires two endpoint conditions. Suppose the initial resource reserve is 5000 units and the owner decides to maintain the object for time period of 10 years. Then the initial condition is $x_0 = 5000$ and final one is $x_T = x_{10} \geqslant 0$ (the final condition is associated with the constraint $g(u_t, t) \geqslant b_t$ from Eq. (3)). It is optimal to extract the whole

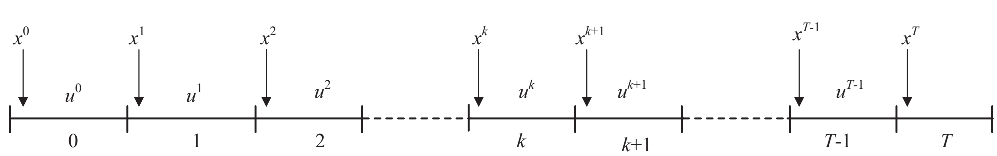

Fig. 1. Overview of state variable x_t and control variable u_t.