International Journal of Mining Science and Technology 25 (2015) 219–223

Contents lists available at ScienceDirect

International Journal of Mining Science and Technology

journal homepage: www.elsevier.com/locate/ijmst

General formulas for drag coefficient and settling velocity of sphere based on theoretical law

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ARTICLE INFO

Article history:
Received 23 July 2014
Received in revised form 7 August 2014
Accepted 21 October 2014
Available online 29 March 2015

Keywords:
Sphere
Drag coefficient
Settling velocity
Reynolds number
Theoretical equation

ABSTRACT

The settlement of particles is of great importance in many areas. The accurate determination of drag coefficient and settling velocity in wide Reynolds number (Re) range remains a problem. In this paper, a series of new formulas for drag coefficient of spherical particles based on theoretical laws, such as the Stokes law, the Oseen law, and the Goldstein law, were developed and fitted using 480 groups of experimental data ($Re < 2 \times 10^5$). The results show that the 2nd approximation of a rational function containing only one parameter can describe C_D–Re relationship accurately over the whole Re range of $0–2 \times 10^5$. The new developed formulas containing five parameters show higher goodness over wide Re range than presently existing equations. The introduction of the Oseen law is helpful for improving the fitting goodness of the empirical formulas. On the basis of one of the Oseen-based C_D–Re formulas giving the lowest sum of squared relative errors Q over the whole Re range ($Re < 2 \times 10^5$), a general formula for settling velocity u_t based on dimensionless parameters was proposed showing high goodness.

1. Introduction

The settlement of particles was involved in many fields such as chemical engineering, water conservancy, environmental engineering, and mineral processing. Despite the extended experimental research on particles of different shapes involving spheres, cuboids, and cylinders, the settlement of the sphere is still the basis of all settlement issues [1–4].

The relationship between the settling velocity and the drag coefficient of a sphere can be obtained in terms of the balance of the gravity and the drag. But the calculation of it is always not an easy one because the drag coefficient (C_D) is not a constant over a wide range of Reynolds number (Re) and there is no definite formula for the calculation of C_D. There are three classical theoretical laws for the calculation of C_D: the Stokes law, the Oseen law, and the Goldstein law. But these laws are only applicable for limited Re. As far as the whole region of $Re < 2 \times 10^5$ is concerned, none of these laws could agree with the experimental data precisely.

Among the formulas for description of C_D–Re, empirical formulas are much more widely used than theoretical ones. These empirical formulas have varying complexity and contain many constants. Many of these correlations are listed in the literatures of Clift et al., Khan and Richardson, and Haider [5–7]. With the improvement of computer performance and data processing, some empirical formulas of the drag coefficient and the settling velocity over wider Re have been further developed. This makes it possible to estimate drag coefficient or settling velocity more precisely approaching experimental values.

Certainly, empirical formulas are not independent of the traditional theoretical laws. These empirical formulas have some characteristics in common: most of them are based on the Stokes law, such as those proposed by Flemmer and Banks, Turton and Levenspiel, Brown and Lawler, and Cheng [8–11]. Yet several scholars proposed formulas based on the Oseen law, such as Dou [12], showing higher goodness than those based on the Stokes law.

On the basis of the Stokes law, the Oseen law, and the Goldstein law for drag coefficient, this article intends to construct new formulas of C_D with higher goodness and applicable for wider Re range as well, and then develops an easily calculated general formula for settling velocity over the wide Re range.

2. Formulas for drag coefficient

2.1. Theoretical law

The theoretical law for the relationship between drag coefficient C_D and Re was obtained by solving the general differential equation of the Navier–Stokes equations.

The Stokes law [13] as shown in Eq. (1) was obtained by neglecting the effects of the inertia term of the Navier–Stokes

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http://dx.doi.org/10.1016/j.ijmst.2015.02.009
2095-2686/

equations. It agrees well with the experimental data only in a small region of $Re < 0.4$.

$$C_D = \frac{24}{Re} \tag{1}$$

The Oseen law [14] linearized the inertia term of the Navier–Stokes equations:

$$C_D = \frac{24}{Re}\left(1 + \frac{3}{16}Re\right) \tag{2}$$

The Goldstein law [15] was developed into more terms and has the following expression for the first five orders (Eq. (3)):

$$C_D = \frac{24}{Re}\left(1 + \frac{3}{16}Re - \frac{19}{1280}Re^2 + \frac{71}{20480}Re^3 - \frac{30179}{34406400}Re^4 + \cdots\right) \tag{3}$$

Both the Oseen law and the Goldstein law are valid only for $Re \leqslant 2$.

A drag coefficient formula of 10th-order approximation suggested by Liao [16] agrees better with the experimental data in a region of $Re < 30$ that is broader than the previous theoretical laws. But the construction of this equation is quite complicated and cannot be expressed simply.

2.2. Empirical formulas

The broad need for more precise estimation of C_D over the whole $Re < 2 \times 10^5$ region has urged the emergence of more empirical formulas. These formulas are developed striving for not only higher fitting goodness to the experimental data but also wider Re range applicable. The most usual formulas for drag coefficient and the corresponding applicable Re range were listed in Table 1.

All these formulas fall into two groups, those of multiplication types, and those of addition types. The multiplication formulas are constructed on the basis of the Stokes law by multiplying a power function or an exponential function, such as Eq. (4) [8]. The addition formulas are always constructed on the basis of the Stokes law by adding a growing function, such as Eqs. (5)–(7) [9–11]. Dou [12] constructed an addition-type formula (Eq. (8)) by adding a growing function on the basis of the Oseen law. Among all these empirical formulas, Eq. (7) is relatively new and showed higher goodness. Thus it is taken here as a comparison.

The empirical formulas really facilitate the estimation of C_D for $Re < 2 \times 10^5$, but the fitting goodness still needs to be improved. Despite the fact that theoretical laws are limited to very small Re range, they are still the basis of all the empirical formulas for C_D. Any empirical formulas for C_D should be based on these classical theoretical laws.

3. Construction of new formulas for C_D based on the theoretical laws

3.1. Constitution of new formulas based on the Stokes law

On the basis of the Stokes law, multiplication and addition methods were both used to construct new formulas. The multiplication formula and the additional formula based on the Stokes law are written as Eqs. (9) and (10):

$$C_D = \frac{24}{Re} f_1(Re) \tag{9}$$

$$C_D = \frac{24}{Re} f_2(Re) + f_3(Re) \tag{10}$$

where $f_1(Re)$, $f_2(Re)$, $f_3(Re)$ are all empirical expressions: $f_1(Re)$ is a power exponent function, $f_2(Re)$ is a polynomial function, and $f_3(Re)$ is a growing function in laminar boundary layer and has a maximum of about 0.47. For $Re = 0$, there are $f_1(Re) = 1$, $f_2(Re) = 1$ and $f_3(Re) = 0$.

Generally, the number of parameters in these expressions is taken to be five, the same as the most empirical formulas for facilitating comparison.

3.2. Constitution of new formulas based on the Oseen law

Based on the Oseen law, multiplication and addition methods were again both used to construct new formulas for C_D. The multiplication equation and the additional equation based on the Oseen law are shown as Eqs. (11) and (12), respectively:

$$C_D = \frac{24}{Re}\left(1 + \frac{3}{16}Re\right) f_1(Re) \tag{11}$$

$$C_D = \frac{24}{Re}\left(1 + \frac{3}{16}Re\right) f_2(Re) + f_3(Re) \tag{12}$$

where $f_1(Re)$, $f_2(Re)$, $f_3(Re)$ are all empirical expressions as mentioned above. The number of parameters involved is five, too.

3.3. Constitution of new formulas based on the Goldstein law

The Goldstein law can be regarded as an approximation by polynomial to an uncertain drag coefficient function. Approximation by polynomial is simple for calculation. But it only agrees to the practical function value in a certain Re range and may cause big error and even diffusion for Re out of this range. To this end, rational function (Eq. (13)) is introduced to approximate the Goldstein law to construct new formulas for C_D.

Table 1
Summary of some empirical formulas.

Author	Formula	Applicable Re	Eq.
Flemmer and Banks	$C_D = \frac{Re}{24} 10^{0.261Re^{0.369} - 0.105Re^{0.431} - \frac{0.124}{1+(\log Re)^2}}$	$Re < 8.6 \times 10^4$	(4)
Turton and Levenspiel	$C_D = \frac{24}{Re}\left(1 + 0.173Re^{0.657}\right) + \frac{0.413}{1+16{,}300Re^{-1.09}}$	$Re < 2 \times 10^5$	(5)
Brown and Lawler	$C_D = \frac{24}{Re}\left(1 + 0.15Re^{0.681}\right) + \frac{0.407}{1+8710Re^{-1}}$	$Re < 2 \times 10^5$	(6)
Niansheng Cheng	$C_D = \frac{24}{Re}(1 + 0.27Re)^{0.43} + 0.47[1 - e^{(-0.04Re^{0.38})}]$	$Re < 2 \times 10^5$	(7)
Guoren Dou	$C_D = \frac{24}{Re}(1 + \frac{3}{16}Re)\cos^3\theta + 0.45\sin^2\theta$ $\theta = \begin{cases} 0, & Re \leqslant 0.5 \\ \frac{\ln 2Re}{\ln 5000}\frac{\pi}{2}, & 0.5 \leqslant Re \leqslant 2500 \\ \frac{\pi}{2}, & 2500 \leqslant Re \end{cases}$	$Re < 2 \times 10^5$	(8)