International Journal of Mining Science and Technology 24 (2014) 451–459

Contents lists available at ScienceDirect

International Journal of Mining Science and Technology

journal homepage: www.elsevier.com/locate/ijmst

A linear programming model for long-term mine planning in the presence of grade uncertainty and a stockpile

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ARTICLE INFO

Article history:
Received 11 September 2013
Received in revised form 7 November 2013
Accepted 15 February 2014
Available online 10 June 2014

Keywords:
Production scheduling
Geostatistics
Conditional simulation
Kriging
Cost of uncertainty

ABSTRACT

The complexity of an open pit production scheduling problem is increased by grade uncertainty. A method is presented to calculate the cost of uncertainty in a production schedule based on deviations from the target production. A mixed integer linear programming algorithm is formulated to find the mining sequence of blocks from a predefined pit shell and their respective destinations, with two objectives: to maximize the net present value of the operation and to minimize the cost of uncertainty. An efficient clustering technique reduces the number of variables to make the problem tractable. Also, the parameters that control the importance of uncertainty in the optimization problem are studied. The minimum annual mining capacity in presence of grade uncertainty is assessed. The method is illustrated with an oil sand deposit in northern Alberta.

1. Introduction

Mine planning is an important process in mining engineering that aims to find a feasible block extraction schedule that maximizes net present value (NPV). In the case of open pit mines, Whittle defines mine planning as: "Specifying the sequence of blocks extraction from the mine to give the highest NPV, subject to variety of production, grade blending and pit slope constraints" [1]. Technical, financial and environmental constraints must be considered.

The uncertainty of the ore grade may cause discrepancies between planning expectations and actual production [2–4]. Various authors present methodologies to account for grade uncertainty, and demonstrate its impact. Dowd proposed a risk-based algorithm for surface mine planning [5]. A predefined distribution function is used for some variables such as commodity price, mining costs, processing cost, investment required, grade and tonnages. Different schedules are generated for a number of realizations of the grades. The proposed method leads to multiple schedules reflecting the grade uncertainty. Ravenscroft and Koushavand and Askari-Nasab used simulated orebodies to show the impact of grade uncertainty on production scheduling [4,6]. They used simulated orebody models one at a time in traditional optimization methods; however, this sequential process does not optimize accounting for uncertainty. Ramazan and Dimitrakopoulos suggested a mixed integer linear programming (MILP) model to maximize NPV for each realization. Then, the probability of extraction of a block at each period is calculated. These probabilities are used in a second stage of optimization to arrive at one schedule. The uncertainty is not used directly in the optimization process [7]. Godoy and Dimitrakopoulos and Leite and Dimitrakopoulos presented a new risk-inclusive long term production plan (LTPP) approach based on simulated annealing [8,9]. A multistage heuristic framework was presented to generate a schedule that minimizes the risk of deviations from production targets. The authors reported a significant improvement in NPV in the presence of uncertainty; however heuristic methods do not guarantee the optimality of the results. Also, these techniques can be difficult to implement, and many parameters may need to be chosen in order to get reasonable results. Dimitrakopoulos and Ramazan presented a linear integer programming (LIP) model to generate optimal production schedules [10]. Multiple realizations of the block model are considered. This model has a penalty function that is the cost of deviations from the target production and is calculated based on the geological risk discount rate (GRD), which is the discounted unit cost of deviation from target production. They use linear programming to maximize a new function that is NPV less penalty costs. It is not clear how to define the GRD parameter.

The shortcomings of the current mine planning methods include: (1) most of the methods show the effect of uncertainty on the mine plan, but do not suggest a method to minimize the risk of uncertainty, (2) the methods minimize the risk or maximize NPV without using uncertainty explicitly, (3) the methods are not suitable for real-size mining problems, (4) there is no methodology

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http://dx.doi.org/10.1016/j.ijmst.2014.05.006

to easily calculate the cost of uncertainty, and (5) none of the presented methods generate an optimum plan in presence of grade uncertainty.

In this paper, a mathematical programming formulism for long term mine planning in presence of grade uncertainty is proposed. The cost of uncertainty is quantified and used in a mixed integer linear programming model. A stockpile is considered in this new model. The cost of uncertainty is needed to determine the optimal trade-off between maximizing the NPV and minimizing the risk of grade uncertainty. The relationship between mining capacity and processing capacity and the cost of uncertainty is shown in this paper.

2. Cost of uncertainty

Typically, the main objective of long-term mine planning is to maximize the NPV of a project subject to technical and other constrains. The goal is to find the sequence of extraction of blocks or mining-cuts. A secondary objective is to account for uncertainty. Recently some authors, such as Dimitrakopoulos and Ramazan, have presented optimization algorithms that aim to maximize NPV and to minimize the negative effects of uncertainty [10]. These methods defer the extraction of more uncertain blocks. In this way the effect of grade uncertainty could be reduced by new information acquired during mining. The key idea is that uncertainty may incur a cost and should be deferred. There are two main costs related to uncertainty:

(1) Cost of under production: where the mine may have to react quickly to make up for an unexpected shortfall.
(2) Cost of over production: unexpected extra ore available to mine may lead to sub optimal use of resources or a cost for stockpiling.

The cost of under production can be assumed the loss of revenue of tonnage of ore that may not be fed to the processing plant and causes the mine and processing plant to operate sub-optimally. A simple method to calculate the cost of under production is:

$$\text{discounted cost of under production} = \text{tonnage of shortfall} \times (\text{average revenue per tonne} - \text{processing cost per tonne})$$

This equation can be rewritten to calculate the discounted cost of under production for period t:

$$C_{up}(t) = T_{up}(t) \times \left(\bar{g}(t) \times \frac{P}{(1+IR)^t} - \frac{C_p}{(1+IR)^t}\right) = T_{up}(t) \times c_{up}(t) \tag{1}$$

where $C_{up}(t)$ is the discounted cost of under production; $T_{up}(t)$ the tonnage of under produced ore in period t; $\bar{g}(t)$ the average input grade to the mill in period t; P the commodity price; C_p the cost of processing per tonne and IR the interest rate. $c_{up}(t)$ is called the discounted cost of underproduction per tonne in period t and it is calculated by Eq.(2) as below:

$$c_{up}(t) = \bar{g}(t) \times \frac{P}{(1+IR)^t} - \frac{C_p}{(1+IR)^t} \tag{2}$$

This approximation for the cost of under production assumes that under production will lead to a loss of revenue due to the mill running at lower capacity. In practice, it is highly likely that the mine will make up the shortfall somehow; however, there is no doubt that under production will incur a cost.

Regarding the cost of over production, there are different components involved. Deferring the extraction of extra ore to the next period entails that the processed ore will have less value due to discounting. The discounting factor also applies to the processing costs. A cost of stockpiling may also be required. The cost of over production could be written as:

$$\text{discounted cost of overproduction} = \text{overproduced ore tonnage} \times (\text{lost of value of ore due to processing in next period} + \text{cost of stockpiling and rehandling})$$

The equation below is proposed to calculate the discounted cost of over production:

$$C_{op}(t) = T_{op}(t) \times \left[\underbrace{\left(\frac{\bar{g}(t) \times P}{(1+IR)^t} - \frac{\bar{g}(t) \times P}{(1+IR)^{t+1}}\right)}_{\text{the lost of the value of ore}} + \underbrace{\left(\frac{C_p}{(1+IR)^{t+1}} - \frac{C_p}{(1+IR)^t}\right)}_{\text{the difference of processing costs}} + \underbrace{\frac{C_{RH}}{(1+IR)^t}}_{\text{rehandling cost}} \right] = T_{op}(t) \times \hat{c}_{op,RH}(t) \tag{3}$$

where $T_{op}(t)$ is the tonnage of over produced ore in period t that is going to be processed in later periods; $C_{RH}(t)$ the re-handling cost of stockpile in period t. $\hat{c}_{op,RH}(t)$ is called the adjusted cost per tonne of overproduction in presence of stockpile in period t. This is an approximation for the cost of over production because the mine may be able to adapt dynamically to the extra ore and divert mining capacity to other locations; nevertheless, there is a cost associated with having more ore available than planned. Therefore it is clear that the cost of over production should be much less than under production in real life. This fact is considered in over production cost calculations. It is assumed that any possible over produced ore that has been transferred to the stockpile will be processed in the next periods that there is a shortfall form target production. Therefore, any cost of over production only is related to losing value of ore due to processing of extra ore in the next period and some stockpiling costs.

The discounted cost of uncertainty in period t over all L realizations is presented in Eq. (4):

$$C_u(t) = \frac{1}{L}\sum_{l=1}^{L}[C_{up}(t;l) + C_{op}(t;l)]$$
$$\bar{C}_u(t) = \frac{1}{L}\sum_{l=1}^{L}[T_{up}(t;l) \times c_{up}(t) + T_{op}(t;l) \times c_{op}(t)] \tag{4}$$

The Discounted Cost of Uncertainty (DCoU) is calculated as in Eq. (5):

$$DCoU = \sum_{t=1}^{T} C_u(t) \tag{5}$$

This gives a single value for the discounted cost of uncertainty over all periods and realizations. It can be used to compare different production schedules. It gives a quantitative measurement for the effect of the grade uncertainty on the long-term production plan. One should note that the cost of underproduction is calculated over all periods except the final period; because any ore that is left for the final period will be processed and will not exceed the target production, any shortfall in the final period is not relevant to the grade uncertainty; therefore $c_{up}(T) = 0$, where T is the final period or the mine life.

3. MILP formulation based on grade uncertainty with stockpile

The MILP model described in Askari-Nasab et al. will be used [11,12]. This model was generalized from an earlier model presented by Caccetta and Hill that is widely accepted [13].