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An improved model of partition curve based on accumulation normal distribution function

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ABSTRACT

Extensive studies based on partition curve of gravity separation have been investigated. All created models are merely used to simulate density distribution at the same size fraction. However, they cannot be used to predictive distribution of materials depending on compound feature of density and size. According to this situation, an improved model of partition curve based on accumulation normal distribution, which was distinguished from conventional model of accumulation normal distribution for partition curve, was proposed in this paper. It could simulate density distribution at different size fractions by using the density–size compound index and conflating the partition curves at different size fractions as one partition curve. The feasibility of three compound indexes, including mass index, settlement index and transformation index, were investigated. Specific forms of the improved model were also proposed. It is found that transformation index leads to the best fitting results, while the fitting error is only 1.75 according to the fitting partition curve.

1. Introduction

Partition curve, the basic curve in the process of gravity separation, is the basis of prediction. It is an important problem in the simulation of gravity separation process to study the characteristics and mathematical models of partition curve [1]. Many researchers have done a lot of studies on the partition curve. The partition curve, the establishment method and the application of the normal integral model have been applied in the process design of coal preparation plant, and also have become the base of computer simulation of partition curve since 1967. With the application of the computer, some foreign researchers began to use the empirical model of the partition curve and got the empirical function of partition curve with the least square method [2,3]. The standard of evaluating gravity separation equipment's process effect was developed in 1986 [4]. The method of fitting the empirical model of the partition curve was systematically introduced in 1991 [5]. Fan et al. used a special Weber distribution model to replace the old normal one to fit the partition curve and achieved a good fitting degree in 1992 [6]. In 1997, Fan et al. did some research about the relationship between multi-hyperbolic tangent parameters and the separating density, and put forward the relation and general formula about the parameters in the model and the separating density in different parallel moving conditions [7]. Song et al. added a deviation coefficient C on the basis of normal model and proposed the partial normal mathematical model of partition curve in coal preparation [8]. In 2005, in order to solve the problem that the characteristic parameters in the special partition curve could not evaluate the separation properties of coal preparation equipment objectively, Chen et al. raised three technical ways to get the average density of high density segment and provided the empirical values of the average density in different high density segments according to the testing data statistical result of gravity separation process's performance evaluation [9]. Based on the form of special partition curve, he proposed the method of combining the special partition coefficient and product partition coefficient predicted to draw the special partition curve. All these studies focused on the accuracy of the partition curve's mathematical model, which could only simulate the material distribution of one single particle size in each density but could not predict that according to the comprehensive features of size–density.

2. Conventional accumulation normal distribution model of partition curve

Conventional accumulation normal distribution model of partition curve is based on accumulation normal distribution function, using probable error E_p (or mechanical error I) and separation density δ_p as model parameters to fit the partition curve [10,11]. The value of E_p, I, δ_p can be got from the actual partition curve through finding the data points and then solving them. And

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$$E_p = \frac{\delta_{75} - \delta_{25}}{2} \tag{1}$$

$$I = \frac{E_p}{\delta_p - 1} \tag{2}$$

$$\delta_p = \delta_{50} \tag{3}$$

where δ_{75}, δ_{50}, δ_{25} are the corresponding density values when the distribution rates are 75%, 50% and 25% in the actual partition curve; E_p and I the deformation degree of the normal distribution integral function; and δ_p shows the translation degree of that [12].

Given the probable error E_p (or mechanical error I), the separation density δ_p of the separation equipment do coordinate transformation.

For the separation process that using E_p as the evaluating index of the separation equipment's separating effect, there is

$$t = \frac{0.6745}{E_p}\left(\delta - \delta_p\right) \tag{4}$$

For the separation process that using I as the evaluating index of the separation equipment's separating effect, there is

$$t = \frac{0.6745}{\log\left(I + \sqrt{I^2 + 1}\right)} \log\left(\frac{\delta - 1}{\delta_p - 1}\right) \tag{5}$$

Put t into the accumulation normal distribution function Eq. (6) and the partition coefficient can be calculated.

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{t^2}{2}} dt \tag{6}$$

But it is complicated to calculate the partition coefficient by using the accumulation normal distribution function directly and the common way is to use Taylor series to do the approximate calculation [1,13]. As

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \frac{x^{10}}{5!} + \ldots \tag{7}$$

Make $x = \frac{t}{\sqrt{2}}$, and Eq. (6) can be transformed into

$$\begin{aligned} f(x) &= \tfrac{1}{\sqrt{\pi}} \int_{-\infty}^{x} e^{-x^2 dx} = \tfrac{1}{\sqrt{\pi}} \int_{-\infty}^{0} e^{-x^2 dx} + \int_{0}^{x} e^{-x^2 dx} \\ &= 0.5 + \tfrac{1}{\sqrt{\pi}} \int_{0}^{x} \left(1 - x^2 + \tfrac{x^4}{2!} - \tfrac{x^6}{3!} + \tfrac{x^8}{4!} - \tfrac{x^{10}}{5!} + \tfrac{x^{12}}{6!} - \tfrac{x^{14}}{7!} + \tfrac{x^{16}}{8!} - \ldots\right) dx \\ &\approx 0.5 + 0.5641895\left(x - \tfrac{x^3}{3} + \tfrac{x^5}{10} - \tfrac{x^7}{42} + \tfrac{x^9}{216} - \tfrac{x^{11}}{1320} + \tfrac{x^{13}}{9360} - \tfrac{x^{15}}{75600} + \tfrac{x^{17}}{685440}\right) \end{aligned} \tag{8}$$

Eq. (8) is the accumulation normal distribution model of the conventional partition curve.

Take the materials that are above 0.074 mm from separation equipment for coarse slime as research subjects. Tables 1–3 show the size and density composition of feed coal, clean coal and waste coal from the equipment.

The partition curve of every particle size is drawn after organizing and calculating the data above (see Fig. 1), with the data from which we can calculate the possible deviation E_p values of every particle size listed in Table 4.

From Table 4, it can be seen that in order to evaluate or predict the separation process of the equipment, four sets of data are needed obviously. But, among them, we cannot find out the data of 0.125–0.074 mm so the E_p and δ_p of this size cannot be got. Therefore, there should be eight model parameters to fit the curve with accumulation normal distribution model and there is no initial model parameter value of the material partition curve in the 0.125–0.074 mm segment. So that it would be easy if there is an improved model that can fit the density distribution of each particle size with one curve and not many model parameters are needed. The author put forward the improved normal distribution integral model of partition curve through finding composite index of the particle size and density.

Table 1
Density and size composition of feed.

Density fraction (g/cm^3)	Size fraction (mm)				
	1–0.5	0.5–0.25	0.25–0.125	0.125–0.074	+0.074
1.2–1.3	0.09	1.49	0.79	0	2.37
1.3–1.4	21.27	14.17	13.35	4.67	53.47
1.4–1.5	7.76	5.84	5.14	3.01	21.75
1.5–1.6	1.45	1.00	0.99	0.74	4.18
1.6–1.8	0.88	0.67	0.42	0.30	2.27
1.8–2.0	0.47	0.28	0.30	0.25	1.31
2.0–2.4	3.43	3.77	5.00	2.46	14.66
Total	35.35	27.23	25.99	11.43	100.00

Table 2
Density and size composition of clean coal.

Density fraction(g/cm^3)	Size fraction (mm)				
	1–0.5	0.5–0.25	0.25–0.125	0.125–0.074	+0.074
1.2–1.3	0	2.19	0.69	0	2.89
1.3–1.4	19.28	15.84	16.32	9.53	60.98
1.4–1.5	5.33	6.72	5.56	7.06	24.67
1.5–1.6	0.42	1.06	1.22	0.87	3.57
1.6–1.8	0.07	0.52	0.69	0.57	1.84
1.8–2.0	0.00	0.07	0.31	0.33	0.71
2.0–2.4	0.00	0.03	1.53	3.79	5.35
Total	25.10	26.43	26.32	22.15	100.00

Table 3
Density and size composition of tailing.

Density fraction (g/cm^3)	Size fraction (mm)				
	1–0.5	0.5–0.25	0.25–0.125	0.125–0.074	+0.074
1.2–1.3	0	0	0	0	0
1.3–1.4	7.01	0.08	0.10	0.00	7.19
1.4–1.5	15.25	0.03	0.04	0.00	15.32
1.5–1.6	7.71	0.05	0.01	0.07	7.85
1.6–1.8	6.27	0.36	0.01	0.03	6.66
1.8–2.0	2.03	1.19	0.03	0.01	3.27
2.0–2.4	17.62	24.79	15.92	1.37	59.70
Total	55.90	26.51	16.11	1.48	100.00

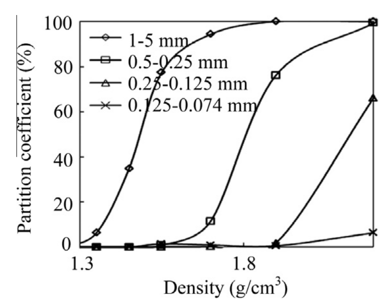

Fig. 1. Partition curves of different size fraction.

Table 4
Values of E_p and δ_p of different size fraction (mm).

Index	Particle size range			
	1–0.5	0.5–0.25	0.25–0.125	0.125–0.074
E_p	0.06	0.08	0.09	
δ_p	1.49	1.82	2.10	