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Effective behavior of a microscopically damaged interface between a layer and a half-space occupied by dissimilar piezoelectric media under antiplane deformations

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A R T I C L E I N F O

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A B S T R A C T

The present paper examines the effective macroscopic behavior of a microscopically damaged interface between an infinitely long piezoelectric layer and a piezoelectric half-space under antiplane deformation. The interface is modeled as containing a periodic array of micro-cracks. The lengths and the positions of the micro-cracks on a period interval of the interface are randomly generated. The micro-statistical model is formulated in terms of hypersingular integral equations and used to investigate in detail the influences of the material constants of the piezoelectric layer and the half-space and the width of the layer on the effective properties of the interface.

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1. Introduction

The macro-level generalized spring-like model for a weak interface Γ between two piezoelectric materials denoted by 1 and 2 is given by the interfacial conditions

$$\left.\begin{array}{l}\underline{\underline{\sigma}}^{(1)}\bullet\underline{\mathbf{n}}=\underline{\underline{\sigma}}^{(2)}\bullet\underline{\mathbf{n}}=\underline{\underline{\mathbf{a}}}\bullet(\underline{\mathbf{u}}^{(1)}-\underline{\mathbf{u}}^{(2)})+\underline{\mathbf{b}}(\phi^{(1)}-\phi^{(2)})\\ \underline{\mathbf{D}}^{(1)}\bullet\underline{\mathbf{n}}=\underline{\mathbf{D}}^{(2)}\bullet\underline{\mathbf{n}}=\underline{\mathbf{c}}\bullet(\underline{\mathbf{u}}^{(1)}-\underline{\mathbf{u}}^{(2)})+h(\phi^{(1)}-\phi^{(2)})\end{array}\right\}\text{ on }\Gamma, \quad (1)$$

where $\underline{\mathbf{n}}$ is the unit normal vector to Γ pointing into material 1, $\underline{\mathbf{u}}^{(i)}$ and $\underline{\underline{\sigma}}^{(i)}$ are respectively the displacement and the stress in material *i*, $\phi^{(i)}$ and $\underline{\mathbf{D}}^{(i)}$ are respectively the electrical potential and the electrical displacement in material *i* and the scalar *h*, the vectors $\underline{\mathbf{b}}$ and $\underline{\mathbf{c}}$ and the second rank tensor $\underline{\underline{\mathbf{a}}}$ are tensorial quantities characterizing the effective properties of Γ.

The spring-like model has been proposed and used by many researchers for analyzing weak interfaces in elastic layered materials (Benveniste and Miloh, 2001; Hashin, 1991; Jones and Whittier, 1967; López-Realpozo et al., 2011; Pilarski and Rose, 1988 and Rokhlin and Wang, 1991) as well as in piezoelectric layered materials (Li and Lee, 2009a, 2009b, 2010; Wang and Pan, 2007; Wang et al., 2007 and Wang and Sudak, 2007). For the case of piezoelectric layered materials, most (if not all) of the existing papers made the assumption that no coupling exists between the displacement and electrical potential jumps in the interfacial conditions, that is, they assumed that $\underline{\mathbf{b}}=\underline{\mathbf{0}}$ and $\underline{\mathbf{c}}=\underline{\mathbf{0}}$ in (1). The validity of such an assumption may be checked by using micro-models to estimate the effective properties of microscopically damaged interfaces.

There are, however, few research papers on micro-analyses for estimating the effective properties of micro-damaged interfaces between dissimilar materials. Fan and Sze (2001) studied the effective electrical behavior of a micro-cracked interface between dielectric materials by using a finite-element based three-phase model. More recently, Wang et al. (2012, 2014, 2015) proposed a micro-statistical model for estimating the effective stiffness of a micro-damaged interface between dissimilar materials under elastostatic deformations.

The current paper adopts the micro-statistical approach to analyze the effective properties of a micro-damaged interface between an infinitely long piezoelectric layer and a piezoelectric half-space under antiplane deformation. As in Wang et al. (2012, 2014, 2015), the interface is modeled as containing a periodic array of micro-cracks which are taken to be either electrically impermeable or permeable. The lengths and the positions of the micro-cracks on a period interval of the interface are randomly generated. The boundary conditions on the micro-cracks are expressed in terms of hypersingular integral equations which are solved numerically. Once the hypersingular integral equations are solved, quantities describing the effective properties of the interface can be readily estimated. The influences of the material constants of the piezoelectric layer and the half-space and the width of the layer on the effective properties of the interface are examined in detail.

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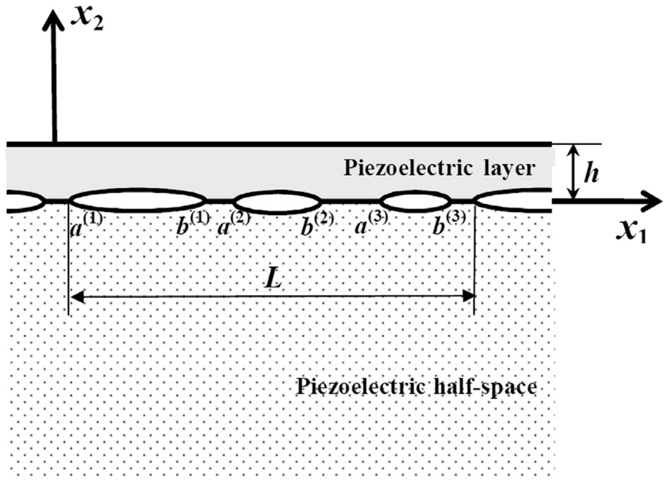

Fig. 1. A sketch of the geometry of the piezoelectric bimaterial for $M = 3$.

The problem under consideration here may be of practical interest as piezoelectric thin film structures are widely used in microelectronics (Park et al., 2014, 2010 and Trolier-McKinstry and Muralt, 2004). Such a structure is formed by coating a thin layer of piezoelectric material on a substrate of dissimilar material (see Tateyama et al., 2009). The interface between the thin layer and the substrate may be damaged by a distribution of micro-cracks. For a simpler mathematical analysis of the layered piezoelectric structure, the interface may be modeled using (1). Unless the edge of the layer is very far away from the micro-cracks, its effects on the effective properties of the micro-cracked interface cannot be ignored in the modeling of the interface.

2. The problem and basic equations

With reference to a Cartesian coordinate system $Ox_1x_2x_3$, consider a thin piezoelectric layer occupying the region $0 < x_2 < h$ (h is a positive constant) bonded to a piezoelectric half-space in the region $x_2 < 0$. The layer and the half-space are occupied possibly dissimilar materials. The interface $x_2 = 0$ between the thin layer and the half-space is microscopically damaged. The geometries of the piezoelectric bimaterial are independent of the spatial coordinate x_3.

The micro-damaged interface is modeled as containing a periodic array of micro-cracks. Specifically, a period interval of the interface contains M arbitrarily located micro-cracks of possibly different lengths. In the region $0 < x_1 < L$, $x_2 = 0$, the tips of the m-th micro-cracks are given by $(a^{(m)}, 0)$ and $(b^{(m)}, 0)$, where $a^{(m)}$ and $b^{(m)}$ ($m = 1,\ 2,\ \cdots,\ M$) are real numbers such that $0 < a^{(1)} < b^{(1)} < a^{(2)} < b^{(2)} < \cdot \cdot \cdot < a^{(M)} < b^{(M)} < L$. The micro-cracks on the remaining part of the interface are given by $a^{(m)} + nL < x_1 < b^{(m)} + nL$ for $m = 1, 2, \ldots, M$ and $n = \pm 1, \pm 2, \ldots$, that is, the remaining parts of the interface are periodically distributed replicas of the region $0 < x_1 < L, x_2 = 0$. Refer to Fig. 1 for a geometrical sketch of the piezoelectric bimaterial having three micro-cracks ($M = 3$) over a period interval of the interface.

The damage ratio (or micro-crack density) ρ of the interface is defined by

$$\rho = \frac{1}{L}\sum_{k=1}^{M}(b^{(k)} - a^{(k)}). \tag{2}$$

The piezoelectric bimaterial undergoes an antiplane deformation with electrical poling in the x_3 direction. The only non-zero component of the Cartesian displacement is u_3 which is a function of only x_1 and x_2. The antiplane stresses σ_{3k} and the electric displacements D_k are given by (see, for example, Auld, 1973 and Li and Lee, 2010)

$$\begin{aligned}\sigma_{3k} &= c_{44}(x_2)\frac{\partial u_3}{\partial x_k} + e_{15}(x_2)\frac{\partial \phi}{\partial x_k},\\ D_k &= e_{15}(x_2)\frac{\partial u_3}{\partial x_k} - \epsilon_{11}(x_2)\frac{\partial \phi}{\partial x_k},\end{aligned} \tag{3}$$

where ϕ is the electrical potential which is also a function of only x_1 and x_2, and $c_{44}(x_2)$, $e_{15}(x_2)$ and $\epsilon_{11}(x_2)$ are respectively the elastic moduli, piezoelectric coefficient and dielectric coefficient of the piezoelectric bimaterial given by

$$(c_{44}(x_2), e_{15}(x_2), \epsilon_{11}(x_2)) = \begin{cases}\left(c_{44}^{(1)}, e_{15}^{(1)}, \epsilon_{11}^{(1)}\right) & \text{for } 0 < x_2 < h,\\ \left(c_{44}^{(2)}, e_{15}^{(2)}, \epsilon_{11}^{(2)}\right) & \text{for } x_2 < 0,\end{cases} \tag{4}$$

with $c_{44}^{(n)}, e_{15}^{(n)}$ and $\epsilon_{11}^{(n)}$ ($n = 1, 2$) being suitably given constants.

According to the law of conservation of momentum and the Gauss law of electric flux, the antiplane deformation of the piezoelectric bimaterial is governed by the partial differential equations

$$\begin{aligned}&\frac{\partial^2}{\partial x_k \partial x_k}(c_{44}(x_2)u_3 + e_{15}(x_2)\phi) = 0,\\ &\frac{\partial^2}{\partial x_k \partial x_k}(e_{15}(x_2)u_3 - \epsilon_{11}(x_2)\phi) = 0.\end{aligned} \tag{5}$$

Note that the Einsteinian convention of summing over repeated indices applies for lower case Latin subscripts which run from 1 to 2.

For the antiplane deformation, the generalized spring-like model in (1) for the micro-cracked interface $x_2 = 0$ between the piezoelectric layer and the piezoelectric half-space can be re-written as

$$\begin{bmatrix}\sigma_{32}(x_1, 0^+)\\ D_2(x_1, 0^+)\end{bmatrix} = \begin{bmatrix}\sigma_{32}(x_1, 0^-)\\ D_2(x_1, 0^-)\end{bmatrix} = \begin{bmatrix}k_{11} & k_{12}\\ k_{21} & k_{22}\end{bmatrix}\begin{bmatrix}\Delta u_3(x_1)\\ \Delta\phi(x_1)\end{bmatrix} \quad \text{for } -\infty < x_1 < \infty, \tag{6}$$

where $\Delta u_3(x_1) = u_3(x_1, 0^+) - u_3(x_1, 0^-)$, $\Delta\phi(x_1) = \phi(x_1, 0^+) - \phi(x_1, 0^-)$ and k_{ij} are coefficients characterizing the effective piezoelectric properties of the interface.

The problem of interest here is to estimate the effective properties k_{ij} of the interface by taking into consideration the details of the interfacial micro-cracks.

For the macro-level model in (6), the interfacial micro-cracks are taken to be electrically impermeable. Thus, there is a jump in the electrical potential ϕ across the interface.

However, if the interfacial micro-cracks are electrically permeable, the electrical potential ϕ is continuous on the interface. For an electrically permeable interface, the interfacial conditions in (6) for the generalized spring-like model should be modified to become

$$\left.\begin{aligned}&\sigma_{32}(x_1, 0^+) = \sigma_{32}(x_1, 0^-) = k\Delta u_3(x_1)\\ &D_2(x_1, 0^+) - D_2(x_1, 0^-) = 0\\ &\phi(x_1, 0^+) - \phi(x_1, 0^-) = 0\end{aligned}\right\} \quad \text{for} \quad -\infty < x_1 < \infty, \tag{7}$$

where k is the effective stiffness of the interface to be estimated.

3. Hypersingular integral formulation

For mathematical convenience, we introduce the generalized displacements U_I and stresses S_{Ik} ($I = 1, 2;\ k = 1, 2$)

$U_1 = u_3,\ U_2 = \phi,\ S_{1k} = \sigma_{3k},\ S_{2k} = D_k,$

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