International Journal of Solids and Structures 96 (2016) 64–73

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

The influence of crack-orientation distribution on the mechanical properties of pre-cracked brittle media

Xiaguang Zeng, Yujie Wei*

LNM, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, PR China

A R T I C L E I N F O

Article history:
Received 18 November 2015
Revised 13 June 2016
Available online 16 June 2016

Keywords:
Pre-cracked media
Crack distribution
Extended finite element method (XFEM)
Elastic properties
Strength

A B S T R A C T

Cracks in kerogen-rich shales and other brittle rock-like materials have a tremendous impact on their elastic properties and strength. In this paper, we investigate the effective mechanical properties of shale plates with pre-existing cracks. We employ the extended finite element method (XFEM) to investigate a pre-cracked medium with an elastic, isotropic and brittle shale matrix. We show how the mechanical properties of the orthotropic shale plates are dependent on the crack density and the standard deviation of crack angles. Both the Young's modulus and the Poisson's ratio of the cracked media exhibit a linear dependence on the standard deviation of crack angles, in contrast to the nonlinear dependence of the strength on the angle deviation. Finally, we propose mechanical models to capture the relationship between the mechanical properties and the distribution characteristics of pre-existing cracks in shales. These phenomenological models could be applied to estimate the fracking behavior of shales in engineering practice.

1. Introduction

Shale gas and shale oil are changing the world's energy market. One key technology that exploits these resources is hydraulic fracturing, more commonly known as fracking (Bažant et al., 2014). To achieve effective fracking, engineers must thoroughly understand the macroscopic mechanical properties of kerogen-rich shales, including their elastic properties and fracture strength. Current knowledge about these elastic properties has been obtained from the interpretation of seismic waves propagating through a shale formation or the laboratory mechanical tests of shale drill cores. Ideally, we desire to construct a sound physical connection between the microstructure characteristics of shales and the macroscopic properties of the media, especially the pre-existing cracks in the media. Typically, shales display significant anisotropy (Sarker and Batzle, 2010; Sondergeld and Rai, 2011; Sone and Zoback, 2013a) and are brittle and transversely isotropic materials, with a symmetry axis vertical to their sedimentary plane (Vasin et al., 2013). The elastic anisotropy is partly caused by the preferred orientation of mineral components (Lonardelli et al., 2007). To address this anisotropy, theoretical works have been developed to predict effective elastic properties (Hornby et al., 1994; Lonardelli et al., 2007; Sayers, 2005; Vasin et al., 2013). Because natural fractures are common in shales and generally have a dominant trend (Gale et al., 2007), they largely contribute to the anisotropy (Vernik, 1993). Following fracking in shale formations, complex crack networks may be present together with the natural cracks; this has been regarded as a critical factor for economic or prospective production from shale reservoirs (Gale et al., 2014; Walton and McLennan, 2013). From this aspect, the interaction of fracking with pre-existing cracks in shales has attracted attention from the fields of solid mechanics, geophysics and composite materials in the past decades.

Theoretical methods have been developed to determine the effective elastic moduli of isotropic solids containing randomly orientated cracks, such as the self-consistent method (Budiansky and O'Connell, 1976), the general self-consistent method (Huang et al., 1994), the differential method (Hashin, 1988; Zimmerman, 1985), and the Mori-Tanaka method (Benveniste, 1987). In parallel, numerical methods have been broadly applied to calculate the effective mechanical properties of solids with pre-existing cracks, such as the finite element method (Makarynska et al., 2008; Shen and Li, 2004) and the boundary element method (Huang et al., 1996; Renaud et al., 1996). The differential method provides the closest estimation at low crack density, whereas the generalized self-consistent method or the non-interaction solution is more accurate than the other methods at high crack density.

The aforementioned methods all use a single parameter – the crack density – to characterize the random crack network and

* Corresponding author. Fax: +86-10-8254-3977.
E-mail address: yujie_wei@lnm.imech.ac.cn (Y. Wei).

http://dx.doi.org/10.1016/j.ijsolstr.2016.06.022
0020-7683/

ignore other important effects, such as the crack orientation and crack length distribution. Therefore, existing theories are not applicable when cracks are aligned with orientation distribution. Some approximations have been suggested to calculate the elastic moduli of these types of materials based on the methods for isotropic cracked solids. For example, Hoenig (1979) worked through two special cases, planar transverse isotropy and cylindrical transverse isotropy with circle cracks, and proposed formulas for the elastic moduli. Some authors (Feng et al., 2003; Gurevich, 2003; Huang et al., 1996; Laws and Brockenbrough, 1987; Thomsen, 1995; Wang et al., 2000; Zhan et al., 1999) later investigated similar situations using different methods. Except for in the case of the aligned crack situation, the crack angle effect is neglected, and the cracked solids are then regarded as macroscopic isotropic materials in the mentioned works. To the best of the authors' knowledge, the crack angle effect on the elastic properties was first analyzed by Sevostianov and Kachanov (2001). The authors showed that the scatter of crack orientation has a pronounced effect on the effective properties of plasma-sprayed ceramic coatings. They developed a quantitative characterization for the microstructures of the coatings using a probability density function (Sevostianov et al., 2004). Later, Giordano and Colombo (2007a, 2007b) dealt with a similar situation and derived a theory for the elastic characterization of cracked solids based on a homogenization technique. Kushch et al. (2009) derived a series solution for the effective elastic moduli of anisotropic cracked materials. These works reported the crack angle effect on the elastic moduli at different crack densities. However, their results failed to explicitly and directly clarify the relationship between the mechanical properties and the crack angle distribution parameters.

In addition, the aforementioned works only calculated the effective elastic moduli. For shale fracking design, the effective tensile strength is also a very important mechanical parameter. The fracture strength is affected by many factors, such as pressure (Lin, 1983) and lamination (Mokhtari et al., 2014). A strong correlation between the shale composition and the intact rock strength has been reported, particularly between the organic matter and the strength (Chong et al., 1982; Sone and Zoback, 2013b). Eseme et al. (2007) discovered a logarithmic empirical relation between the tensile strength and temperature. Shales' fracture strength is also reduced drastically by their cracks, which was demonstrated experimentally (Gale and Holder, 2008). Zhang et al. (1998) theoretically investigated the effects of the crack-length distribution and ligament sizes in the case of strongly interacting collinear cracks. Ma et al. (2005) used their numerical method to examine the influences of the crack distribution on the tensile strength. They found that the tensile strength exhibited a pronounced dependence on the distribution of crack orientations and crack locations as well as on the crack density. However, it appears that no study has thus far discovered the relationship between the general characterization of crack angles and the fracture strength for shales. Therefore, there is a compelling need to investigate the crack angle influence on the shales' fracture strength.

In this paper, we examine the effects of crack angles on the mechanical properties of elastic brittle cracked shale plates. We employ numerical simulations to clarify the relationship between the crack angle distribution and the effective Young's modulus, Poisson's ratio, shear modulus and tensile strength. Furthermore, we propose approximation formulas that capture trends revealed in our numerical simulations. Section 2 describes the assumptions employed in this study and the model for effective mechanical properties. Section 3 presents the numerical results from extended finite element method (XFEM) simulations. Section 4 contains final discussions and concluding remarks.

2. Problem description

2.1. Crack distribution

As a representative shale matrix body, we consider an initially isotropic brittle linear elastic plate of area A with Young's modulus E, Poisson's ratio ν and tensile strength σ_s. This plate is permeated by N arbitrarily oriented straight cracks that do not intersect and whose centers are distributed homogenously and randomly without overlapping. Each crack can be characterized by two variables: crack length l_i and crack angle θ_i (the angle between the crack plane and axis x_1), as shown in Fig. 1. In the present work, we assume that these two random variables obey a truncated Gaussian distribution. The crack length l_i and the crack angle θ_i lie within the intervals $l_i \in (0, +\infty)$ and $\theta_i \in (0, \pi)$, respectively. Their probability density functions can be calculated by the following function (Johnson et al., 1995):

$$f(x;\mu,s,c,d) = \begin{cases} \dfrac{\frac{1}{s}\phi\left(\frac{x-\mu}{s}\right)}{\Phi\left(\frac{d-\mu}{s}\right)-\Phi\left(\frac{c-\mu}{s}\right)}, & c<x<d \\ 0, & x\le c, x\ge d \end{cases}, \tag{1}$$

where ϕ is the standard Gaussian probability density function, Φ is the cumulative distribution function, μ is the expectation, s is the standard deviation, and c and d bound the region of interest. We can calculate the expectation and standard deviation of the crack length or the crack angle by the following formulas (Johnson et al., 1995):

$$\mu_t = \mu + \frac{\phi\left(\frac{c-\mu}{s}\right)-\phi\left(\frac{d-\mu}{s}\right)}{\Phi\left(\frac{d-\mu}{s}\right)-\Phi\left(\frac{c-\mu}{s}\right)}s, \tag{2a}$$

$$s_t = s\sqrt{1+\frac{\frac{c-\mu}{s}\phi\left(\frac{c-\mu}{s}\right)-\frac{d-\mu}{s}\phi\left(\frac{d-\mu}{s}\right)}{\Phi\left(\frac{d-\mu}{s}\right)-\Phi\left(\frac{c-\mu}{s}\right)}-\left(\frac{\phi\left(\frac{c-\mu}{s}\right)-\phi\left(\frac{d-\mu}{s}\right)}{\Phi\left(\frac{d-\mu}{s}\right)-\Phi\left(\frac{c-\mu}{s}\right)}\right)^2}, \tag{2b}$$

and herein, the truncated Gaussian distribution random variables describing the cracks are numerically simulated using the method proposed by Chopin (2010).

The assumed distributing cracks can be described by four distribution characteristic parameters, i.e., crack length expectation μ_{tL}, crack length standard deviation s_{tL}, crack angle expectation μ_{tA} and crack angle standard deviation s_{tA}. To simplify the analysis, we should first reduce the variables of the problem. According to the basic definitions of crack density ξ, crack length expectation μ_{tL} and crack length standard deviation s_{tL}, we may connect ξ with μ_{tL} and s_{tL} with the following derivation. The ratio of cracked surface ξ is defined as $\xi = \frac{1}{A}\sum_{i=1}^{N}\left(\frac{l_i}{2}\right)^2$, where A is the area of the cracked plate, N the total number of cracks, and l_i the length of the i-th crack for i=1, …, N. It is straightforward to write the crack length expectation and its standard deviation as $\mu_{tL} = \frac{1}{N}\sum_{i=1}^{N} l_i$ and $s_{tL} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(l_i-\mu_{tL})^2}$, respectively. We may reformulate the definition for s_{tL}, as

$$s_{tL}{}^2 = \frac{1}{N}\sum_{i=1}^{N}(l_i-\mu_{tL})^2 = \frac{1}{N}\sum_{i=1}^{N}\left(l_i^2 - 2l_i\mu_{tL} + \mu_{tL}{}^2\right)$$

$$s_{tL}{}^2 = \frac{1}{N}\sum_{i=1}^{N} l_i^2 - \frac{2\mu_{tL}}{N}\sum_{i=1}^{N} l_i + \frac{1}{N}\sum_{i=1}^{N}\mu_{tL}{}^2.$$