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A large strain hyperelastic viscoelastic-viscoplastic-damage constitutive model based on a multi-mechanism non-local damage continuum for amorphous glassy polymers

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A B S T R A C T

A large strain hyperelastic phenomenological constitutive model is proposed to model the highly nonlinear, rate-dependent mechanical behavior of amorphous glassy polymers under isothermal conditions. A co-rotational formulation is used through the total Lagrange formalism. At small strains, the viscoelastic behavior is captured using the generalized Maxwell model. At large strains beyond a viscoelastic limit characterized by a pressure-sensitive yield function, which is extended from the Drucker-Prager one, a viscoplastic region follows. The viscoplastic flow is governed by a non-associated Perzyna-type flow rule incorporating this pressure-sensitive yield function and a quadratic flow potential in order to capture the volumetric deformation during the plastic process. The stress reduction phenomena arising from the post-peak plateau and during the failure stage are considered in the context of a continuum damage mechanics approach. The post-peak softening is modeled by an internal scalar, so-called softening variable, whose evolution is governed by a saturation law. When the softening variable is saturated, the rehardening stage is naturally obtained since the isotropic and kinematic hardening phenomena are still developing. Beyond the onset of failure characterized by a pressure-sensitive failure criterion, the damage process leading to the total failure is controlled by a second internal scalar, so-called failure variable. The final failure occurs when the failure variable reaches its critical value. To avoid the loss of solution uniqueness when dealing with the continuum damage mechanics formalism, a non-local implicit gradient formulation is used for both the softening and failure variables, leading to a multi-mechanism non-local damage continuum. The pressure sensitivity considered in both the yield and failure conditions allows for the distinction under compression and tension loading conditions. It is shown through experimental comparisons that the proposed constitutive model has the ability to capture the complex behavior of amorphous glassy polymers, including their failure.

1. Introduction

Amorphous glassy polymers are widely used in combination with different types of fibers to manufacture fiber reinforced polymers (FRPs) for applications in a wide range of industrial components. Due to the increasing interest in FRPs, accurate constitutive representations of their constituents are more and more required to accurately capture the mechanical behavior through multiscale analyses. By considering proper constitutive model for each constituent of composite materials, a computational micromechanics approach can be achieved to link up the FRPs behavior to not only constituent behaviors but also to the constituent arrangements as well as the constituent interactions (Canal et al., 2009; Melro et al., 2013, *e.g.*).

In general, the mechanical behavior of amorphous glassy polymers depends on the strain rate, hydro-static pressure and temperature, as demonstrated through numerous experimental tests (Boyce et al., 1994; Buckley et al., 2001; Chen et al., 2002; Fiedler et al., 2001; Hine et al., 2005; Lesser and Kody, 1997; Morelle et al., 2015; Mulliken and Boyce, 2006). A typical stress-strain behavior for this kind of materials under uniaxial monotonic loading conditions is sketched out in Fig. 1(a), in which the whole stress-strain curve can be divided into multiple stages. After an elastic stage

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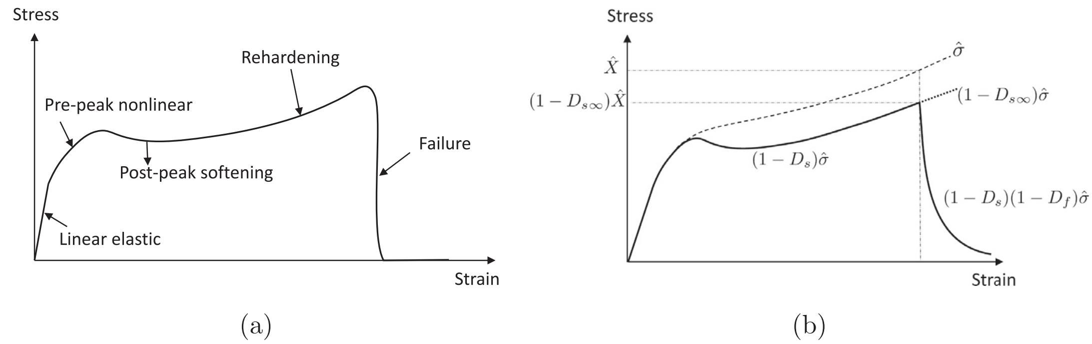

Fig. 1. Typical stress-strain curve of amorphous glassy polymers at a constant given engineering strain rate: (a) multiple stages and (b) modeling strategy.

at small strains, a nonlinear stage continues until reaching a peak stress, where large molecular movements can take place. After this peak value, the stress tends to decrease with increasing deformation. This effect is called softening. The physical origin of softening is still subject to debate but seems related to the kinetics of initiation, growth, and coalescence of shear transformation zones, which translate into the micro-shear banding as a true material feature (Morelle, 2015). At large strains, when the softening is saturated, a rehardening stage takes place until failure is attained. The hardening phenomenon in glassy polymers has been interpreted using the analogy of an entropic spring (the three-dimensional polymer network), of which the non-linear stiffness depends notably either on the density of entanglements for thermoplastics (Boyce et al., 1988; Tervoort et al., 1997; Wu and van der Giessen, 1995) or the density of cross-links for thermosets. The compressive behavior differs from the tensile one as a result of the pressure-dependent yielding of polymers. Larger compressive peak stresses, failure stresses, and failure strains are normally observed in comparison to the tensile case at the same strain rate. The full range behavior shown in Fig. 1(a) is observed with epoxy resins under compressive loading, while under tensile loading the fracture of epoxy resin can occur prematurely before reaching the peak stress (Fiedler et al., 2001; Morelle et al., 2015), so the softening and rehardening phenomena are not always observed. In the case of a ductile, thermoplastic polymer (e.g. polycarbonate), this full range behavior appears in compression, tension, and shear loading conditions (Boyce et al., 1994). At small strain levels, the stiffness increases with the strain rates. A viscoelastic contribution should be taken into account at this stage. At higher strain levels, the non-linearity is enhanced with the presence of plasticity. The stress–strain response exhibits a strong strain rate dependence in the plastic regime. The material microstructure always involves internal defects at nano- and micro-scale. Moreover, the material microstructure can be modified under various loading conditions with the presence of some irreversible phenomena such as cavitation, chain scission *etc.*, leading to a significant amount of micro-voids and micro-cracks. These pre-existing and arising defects can then contribute to the stress decrease in the softening stage and play an important role on the initiation of the failure stage. A couple viscoelastic-viscoplastic-damage constitutive model in the large strain framework is thus necessary in order to capture the entire range of stress-strain responses at various strain rates of these materials.

The rate dependent behavior of polymers in general can be modeled using a viscoelastic constitutive law. Some available viscoelastic models can be used as the generalized Maxwell model (Buhan and Frey, 2011; Reese and Govindjee, 1998), generalized Kelvin model (Zhang and Moore, 1997), fractional model (Schiessel et al., 1995), or Schapery model (Haj-Ali and Muliana, 2004). In these models, a network of multiple springs and dashpots is considered. The rate effect is modeled by a Newtonian fluid flow governing each dashpot. Although these models can be extended to the finite strain regime and consider non-linear Newtonian fluid flows for the dashpots, a proper viscoelastic constitutive model cannot capture the complex behavior of amorphous glassy polymers, which combines different complex mechanisms, such as plasticity, softening, failure ingredients, *etc*. This motivates the use of a more complex constitutive model. For this purpose many viscoplastic models have been proposed to predict the rate dependent behavior of polymers. On the one hand, the physically-based constitutive models have been proposed, see *e.g.* some physically-based models proposed by Arruda et al. (1995); Boyce et al. (1988); Govaert et al. (2000); Tervoort et al. (1997). Although these models can capture the complex behavior of polymers in the glassy state, the experimental calibration of their constitutive parameters can be complex. On the other hand, the phenomenological-based constitutive models have been developed and sometimes provide an easier modeling approach. Most of them were initially used for metals and then extended to polymers. In this category, the Perzyna viscoplasticity theory (Perzyna, 1971) can be used to model the rate-dependent behavior of polymers as demonstrated by Abu Al-Rub et al. (2015); Kim and Muliana (2010); Van Der Sluis et al. (2001); the viscoplastic theory based on the overstress (VBO) concept (Krempl et al., 1986) can be considered for polymers as shown by Colak (2005); the Bodner and Partom viscoplastic model (Bodner and Partom, 1975) can be applied to polymers as also shown by Frank and Brockman (2001); Zaïri et al. (2008). The complex behavior of polymers can thus be captured using a viscoelastic-viscoplastic constitutive model with a robust integration algorithm to be implemented in finite element codes (Miled et al., 2011) by combining a viscoelastic constitutive model with a phenomenological-based viscoplastic one. Additionally, to model the material degradation when dealing with the fracture behavior of polymers, a viscoelastic-viscoplastic-damage constitutive model can be used (Abu Al-Rub et al., 2015; Krairi and Doghri, 2014; Zaïri et al., 2008). However, the complex mechanical behavior of amorphous glassy polymers exhibiting multiple stages coupled with the compression-tension asymmetry in both yielding and failure stages were not considered. The failure of amorphous glassy polymers can be studied with a physically-based constitutive model coupled with a crazing model as considered in Chowdhury et al. (2008a; 2008b).

When dealing with softening phenomena, a continuum damage mechanics (CDM) approach can be used (Kachanov, 2013; Lemaitre and Chaboche, 1994). The material softening is modeled by a set of internal variables (so-called damage variables) in order to capture the local stress reduction. Beyond the onset of softening, the deformation tends to localize into a narrow zone. If a standard continuum is considered when strain localization happens, its underlying local action assumption, in which the stress state at a material point depends only on the deformation state at that point, leads to the loss of solution uniqueness. Consequently, the boundary value problem becomes ill-posed. The numerical solution obtained from