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Buckling analysis, design and optimisation of variable-stiffness sandwich panels

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ABSTRACT

In recent years variable-stiffness (VS) technology has been shown to offer significant potential weight savings and/or performance gains for both monolithic and stiffened plate structures when buckling is a driving consideration. As yet, little work has been reported on VS sandwich structures. As such, a semi-analytical model is developed based on the Ritz energy method for the buckling of sandwich panels with fibre-steered VS face-sheets. The model captures both global and shear crimping instabilities and is shown to explain both types of buckling responses observed and the mode switching between them. Quantitative agreement with detailed three-dimensional finite element analysis was found to be within 13%. Subsequent parametric and optimisation studies, which were performed for many practical geometries using the developed model, reveal that, whilst VS sandwich panels show a significant improvement in global buckling performance, they suffer a reduction in shear crimping performance when compared to their straight-fibre counterparts. This behaviour is found to be due to the VS face-sheets creating a pre-buckling load redistribution where regions locally exceed the critical shear crimping load and induce the short wavelength instability at a reduced panel level load. For VS sandwich panels with modest to low transverse shear moduli of the core, shear crimping can become the critical mode diminishing performance benefits relative to straight-fibre configurations. Cores with sufficient rigidity, thus preventing shear crimping, showed improvements in critical buckling load in the order of 80% when using VS, however this improvement reduces to a negligible amount with decreasing core transverse shear moduli. The transverse shear flexibility and load redistribution are thus two key parameters that must be considered carefully in the design of sandwich panels, in order to exploit the benefits of VS fully in this novel structural configuration.

1. Introduction

Traditional tailoring of fibre-reinforced composites is achieved by varying the orientation of fibres through the thickness of a laminate. However, recent advancements of automated fibre placement (AFP) and tape laying technologies have led to the possibility of steering fibres in the plane of a ply, thus creating variable-stiffness laminates and significantly increasing the design space available to engineers. In recent times, performance benefits of VS laminates have been shown for: the buckling and post-buckling of plates (Groh et al., 2013; IJsselmuiden et al., 2010; Olmedo and Gürdal, 1993; Raju et al., 2015; Weaver et al., 2009; Wu et al., 2013; 2012b), shells (White et al., 2014) and stiffened panels (Coburn and Weaver, 2015; Coburn et al., 2014a; 2014b); the stress distribution around discontinuities (Hyer and Lee, 1991; Khani et al., 2011); and stiffness blending of structures (Panesar et al., 2012; Wu et al., 2009). The manufacture of VS laminates has been achieved largely through the established AFP (Wu et al., 2009) and lately through an early concept manufacturing method called continuous tow shearing (Kim et al., 2012; 2014).

The majority of work to date on the buckling of VS structures is limited to simple configurations, such as plates and shells, with simple boundary conditions. Whilst analytical and semi-analytical methods (Olmedo and Gürdal, 1993; Weaver et al., 2009; Wu et al., 2012b) provide an accurate and computationally efficient alternative to finite element analysis (FEA), they are often limited to simple cases in terms of loading, boundary conditions, geometry and structural configuration. Recently, Coburn et al. (2014a); 2014b) developed an analytical model based on the Ritz energy method for the buckling of stiffened VS panels including a first-order shear deformation theory (FSDT) to enable the study of thick laminate sections. Using the model in a follow-up optimisation study (Coburn and Weaver, 2015), it was shown that stiffened VS panels can

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achieve mass reductions when compared to straight-fibre stiffened panels, albeit to a lesser extent than plates considered in isolation (Gürdal et al., 2008; IJsselmuiden et al., 2010; Olmedo and Gürdal, 1993; Wu et al., 2012b).

One application of VS that has not been explored to date, to the best of the authors' knowledge, is the use of fibre-steering in composite face-sheets to create VS sandwich panels. Sandwich panels are well known to be efficient structures in bending and buckling and for this reason are commonly used in regions where supported edges are spaced far apart. The aim of this work is to use the approach presented by Coburn et al. (2014b), including a FSDT in a semi-analytical Ritz model, to explore structural configurations of VS sandwich panels and the effect of fibre-steering and core shear flexibility on buckling performance. In particular, the study will focus on translating the case identified by Olmedo and Gürdal (1993), Gürdal et al. (2008), who showed that a simply-supported square plate subject to uniform end-shortening can achieve up to 80% improvement in critical buckling load with a lateral direction fibre path variation, to sandwich panels. We found that the simple FSDT was of sufficient fidelity to understand and explain the response of the sandwich panels we consider. Future work could consider development of models, such as that recently developed by Groh and Weaver (2015), which could represent the behaviour of more general sandwich panels that could be thicker or have more complex boundary conditions and loading.

The following sections are structured as follows: Section 2 develops a two-dimensional semi-analytical model for the buckling analysis of VS sandwich panels subject to uniaxial and biaxial loading with clamped and simply-supported boundary conditions applying a FSDT; a parametric study is then performed in Section 3 on the fibre angle variation for panels with various core properties, followed by a discussion of the failure modes and significance of core shear stiffness; in Section 4, an optimisation study with an increased variation in fibre-steering is performed identifying optimal configurations and improvements; and finally, the paper is concluded in Section 5.

2. Semi-analytical model

The semi-analytical model considers uniaxial and biaxial loading of a prismatic sandwich panel with fibre-steering in the lateral direction (y-direction). Previous studies on VS plates (Gürdal et al., 2008) indicate that a one-dimensional variation, perpendicular to the principal loading direction leads to a favourable redistribution of loads towards supported boundaries, improving the global buckling performance and additionally allowing manufacturable fibre paths. The extension of the model to fibre paths varying in the longitudinal direction or in-plane shear loading could be achieved with the use of a more generalised Airy stress function and the pre-buckling boundary conditions as detailed in Wu et al. (2012a).

Transverse shear strains are included in the analysis with a through thickness weighted average approach, as is typically the case with a FSDT. FSDTs (Mindlin, 1951; Reissner, 1944; Timoshenko, 1934) can provide accurate solutions with low computational cost for moderately thick plates and have previously been incorporated into the buckling analysis of thick plates and sandwich structures by applying the Ritz method and assuming additional shape functions for either rotation or transverse shear strain profiles (Dawe and Craig, 1986; Ko and Jackson, 1991; Libove and Batdorf, 1948). By smearing properties and idealising the three-dimensional structure in two-dimensions, the FSDT is suitable for predicting global events such as buckling. For structures exhibiting large variations in stiffness through the thickness, such as sandwich panels, care must be taken when interpreting results obtained from a FSDT. It is noted that for characteristic length to thickness ratios less than typically 20:1, or the requirement for detailed through thickness stresses, the FSDT approach detailed herein could be insufficient and higher-order shear deformation theories or full three-dimensional analyses may be required. The characteristic length of a panel, for the purposes of buckling analyses, is the dimension which drives the size of the buckling half-waves. For the case of long panels with edges free to expand, as is the case for the parametric and optimisation studies presented herein, this is typically the panel's shorter transverse dimension. The characteristic length to thickness ratio provides a rule-of-thumb guide to the importance of through-thickness behaviour, as the characteristic length is a measure of the length over which a buckled half-wave occurs. Thus a more appropriate value for characteristic length, would therefore be the shortest dimension of the buckling half-waves that form, this however requires knowledge of the solution, and for pre-analysis guidance, an appropriate panel dimension is therefore commonly used.

In this study the semi-analytical model was implemented in The MathWorks Inc. (2013).

2.1. Sandwich panel description and loading

The sandwich panel considered consists of two identical VS composite face sheets, individually constrained to be balanced and symmetric about their local geometric midplane ($\mathbf{B}, A_{16}, A_{26} = 0$), either side of a homogeneous orthotropic core material as shown in Fig. 1. The sandwich panel is prismatic in loading, boundary conditions, and properties, with the fibre-steering restricted to the lateral direction of the laminates (y-direction). The panel is subject to uniaxial and biaxial uniform end-shortening.

2.2. Skin fibre path representation

Several definitions for the fibre variation over the plane of a laminate have been proposed in the literature, ranging from simple linear variations (Olmedo and Gürdal, 1993) to more complex higher-order variations (Wu et al., 2012b). Here, the piecewise linear method (Blom et al., 2008) is used due to its ease of visualisation and immunity to Runge's phenomenon (Runge, 1901). The fibre angle in the y-direction is given by:

$$\theta(y) = \begin{cases} \theta_{1,2}(y) & \text{if} \quad y_1 \le y < y_2 \\ \theta_{2,3}(y) & \text{if} \quad y_2 \le y < y_3 \\ \dots & \\ \theta_{n-1,n}(y) & \text{if} \quad y_{n-1} \le y < y_n \end{cases} \tag{1}$$

where

$$\theta_{i,i+1}(y) = \left(\frac{T_i - T_{i+1}}{y_i - y_{i+1}}\right) y + \frac{T_{i+1} y_i - T_i y_{i+1}}{y_i - y_{i+1}};$$

T_i and T_{i+1} are the ith and (ith + 1) control point fibre orientations respectively; and y_i and y_{i+1} are the global coordinate system y-positions of the ith and (ith + 1) control points respectively. A VS ply is designated by $\langle T_1 | T_2 | \dots T_n \rangle$ for n control points whose positions are given by $[y_1, y_2, \dots, y_n]$.

2.3. Formulation overview

The three-dimensional sandwich panel is idealised as a Mindlin-Reissner two-dimensional plate (Mindlin, 1951; Reissner, 1944), i.e. with transverse shear flexibility, as shown in Fig. 1. Prior to the buckling analysis, a pre-buckling analysis is required to determine the non-uniform stress resultant distribution in the VS structure when subject to uniform end-shortening. This is achieved using the approach of Wu et al. (2012b), albeit for the simpler prismatic case, by formulating the total complementary energy of the system and solving for the unknown Airy stress function with the Ritz method. For the buckling analysis, the total potential energy of