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Real-time modelling of diastolic filling of the heart using the proper orthogonal decomposition with interpolation

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ABSTRACT

In this research, a reduced order method (ROM) called the Proper Orthogonal Decomposition with Interpolation (PODI) is used to drastically reduce computation time of highly complex and non-linear problems as encountered in simulating the heart. The idea behind this method is to first construct a database of pre-computed full-scale solutions using the Element Free Galerkin method (EFG) and then project a selected subset of these solutions to a low dimensional space. Using the Moving Least Square method an interpolation is carried out for the problem at hand, before the resulting coefficients are projected back to the original high dimensional solution space. Computations are carried out on a bi-ventricle model to investigate the performance and accuracy varying the material parameters and to determine the sensitivity with respect to the parametric values. The PODI calculations are completed within 1.25 s on a normal desktop machine with the relative ℓ_2 error norm not exceeding 2.5×10^{-3}. Hence, it is demonstrated that real-time modelling of the heart can be successfully carried out at acceptable error levels.

1. Introduction

Computational cardiac mechanics is emerging as a rapidly expanding area of research bringing together multidisciplinary research centred on understanding the physiological and mechanical behaviour of the heart at scales ranging from cell to tissue and organ levels. Principles of continuum mechanics are key in creating a realistic multi-scale model of the heart. They allow one to describe the directly observable behaviour of the heart by incorporating its complex heterogeneous and anisotropic structure as well as the coupling mechanisms between mechanical fields on the one hand and chemical and electrical fields on the other. Computational models therefore help to quantify the bio-mechanical environment in health, injury, and disease which, in turn, leads to advances in diagnostic and therapeutic procedures.

In cardiac modelling, many efforts have been made towards simulating a full heart beat in order to help evaluating the performance of the heart. In the past 30 years, the field of cardiac modelling has undergone major development that has made it possible to be close to a full working mathematical model that can mimic the heart's behaviour in the very realistic way needed to obtain medical performance indicators (Baillargeon et al., 2014). This has been achieved by employing complex non-linear partial and ordinary differential equations which are solved using staggered iterative schemes. However, these calculations are computationally extremely expensive. In Vetter and McCulloch (2000), Walker et al. (2005), Niederer and Smith (2009), and Lafortune et al. (2012), it has been found that the required computational resources can vary from at least 16 to as much as 200 processors on a high performance computing cluster for calculation times ranging between 1–50 h for only one single heart beat. Consequently, practical application of such models has been very limited in the medical field, because the strength of heart modelling would lie in predicting the behaviour and performance of a specific heart under different physiological conditions concerning hemodynamical loads, progressing disease, therapeutical measures or medication etc. For all those scenarios, it is required to run a series of simulations capturing the heart's behaviour over a time period longer than one single heart beat in order to obtain representative results. This, however, cannot be achieved currently by conventional means of computer modelling. Hence, the solution proposed in this research is to use a Reduced Order Method (ROM) and amongst those, specifically, the so-called Proper Orthogonal Decomposition with Interpolation (PODI).

Generally, POD provides the means to extract information from a predefined set of data obtained from a set of experiments or computer simulations using statistical methods. In particular, it has

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the ability to capture important details or trends and represents them in terms of a set of numerical vectors known by several names such as *POD basis, Proper Orthogonal Modes (POM), empirical Eigenfunctions or empirical orthogonal functions*.

The POD method has been developed independently by several researchers. According to Holmes et al. (1996), the first ones to have established the method were Kosambi (1943), Loève (1945), Karhunen (1946), Pougachev (1953), and Obukhov (1954). However, its first application was suggested by Lorenz (1956) for weather prediction. Since then, the POD method has gained great popularity due to its simplicity, flexibility, ease of implementation, ability to reduce the order of a system of equations and application to a broad range of field problems such as:

- **Image processing**
  In Everson and Sirovich (1995), exploited the POMs to recover missing data from the facial images of human beings. Moreover, they also attempted to use the POD basis to reconstruct the face of a monkey. By doing so, the authors managed to prove that the POMs are specific to the data set from which they have been computed as the reconstituted face had similar resemblance to human facial features.
- **Heat flow problem**
  Falkiewicz and Cesnik (2011) showed that is possible to decouple their system of equation making use of the dominant POMs when solving of the heat transfer problem of a hypersonic vehicle through time.
- **Computational fluid dynamics(CFD)**
  In the CFD field, the POD method is well established, as it has been used quite extensively. For example, Amsallem et al. have set-up a database of pre-computed POD-based reduced order information in order to achieve near-real-time simulation of aero-elastic prediction of an aircraft under certain operational conditions. In Christensen et al. (2000), used two modified POD methods, called weighted POD (*w*-POD) and predefined POD (*p*-POD), to demonstrate how the choice of different POMs can affect the calculation of fluid flow with a different Reynolds number. Finally, Druault et al. (2005) employed the POD method to interpolate the time information between two consecutive particle image velocimetry measurements in a spark ignition combustion engine to obtain a smooth change in their solution field.
- **Solid mechanics**
  Several usages of the POD method in solid mechanics frameworks have been found in literature, amongst the first Georgiou and Sansour (1998) who analysed the dynamics of non-linear in-plane rods. Generally, employing of POD in vibration analysis of structures is quite common (Amabili et al., 2006; Georgiou, 2005; Han and Feeny, 2003; Mariani and Dessi, 2012). It can help in understanding the different deformation modes that a structure undergoes before failing or when under impact (Trindade et al., 2005). In Gonalves et al. (2008), POD was employed to reduce the complexity of modelling a cylindrical shell structure by making use of a perturbation procedure. The same strategy was also applied by Brigham and Aquino (2009) to model the inverse viscoelastic material characterization in acoustic-structure interaction.

As mentioned above, this research will utilize the PODI method which is a particular variant of POD. It was developed by Ly and Tran (2001) and involves a collection of datasets of the structure under consideration describing its mechanics for a range of variations in terms of geometry, material properties, loading conditions etc. These datasets are transferred to a low-dimensional space via their associated POD-basis where the unknown or not yet determined mechanical response of a structure of the same category is interpolated. Note this stands in contrast to methods which involves direct interpolation of Proper Orthogonal Modes (Niroomandi et al., 2012) or interpolation of reduced order models in the tangent space (Amsallem et al., 2009). In our case, the collection of datasets comprises of full-scale simulation results of the human heart obtained using the Element-Free Galerkin method (EFG) (Dolbow and Belytschko, 1998) for different cardiac tissue parameters (such as stiffness, fibre orientation, … etc.). After these results are obtained, they are then stored off-line in a suitable database format for ease of access. Recent research carried out by Niroomandi et al. (2008), Coelho et al. (2009), and Ko et al. (2011) have found that such an approach facilitates sub-second calculation times and therefore making high frequency computation feasible (Coelho et al., 2009; Ko et al., 2011; Niroomandi et al., 2008).

Regarding the interpolation technique the Moving Least Square approximation (MLS) is favoured in this work. This is mostly due to its ability to scale up smoothly to several dimensions when multi-parametric simulations will be carried out while still generating accurate results. Studies, such as Coelho et al. (2009), and Ko et al. (2011), have already coupled POD with MLS aiming at capturing localised phenomena in the solutions.

The application of the PODI method to heart modelling will be presented in this paper in the following structure. In Section 2.1, the ROM will be revisited with a particular reference to the POD method. The Proper Orthogonal Decomposition with Interpolation will then be elaborated on in Section 2.2 as a method to circumvent some of the POD difficulties, followed by the Moving Least Square Approximation method, in Section 3, which serves for interpolation purposes in this work. In Section 4, the cardiac mechanics equations along with the bi-ventricular model used for simulation of the heart will then be introduced. Our methodology will then be detailed in Section 5 before we finally present the results of our approach in Section 6.

2. Reduced order method

A reduced Order Method (ROM) is a technique commonly used to decrease the complexity of large system of equations. This is achieved by compressing the whole system to such a point that accuracy is not greatly reduced and that the general behaviour of the problem, e.g. its mechanics, is preserved. One widely used method classified as ROM is the so-called *Proper orthogonal decomposition* which will be utilised in this research to achieve real-time modelling of the heart.

2.1. Proper orthogonal decomposition

POD can be used to extract features from any dataset consisting of either linear or non-linear data. In the literature, it is usually found in the form of the Kharhunen–Loève Decomposition (KLD), the Singular Value Decomposition (SVD) or the Principal Component Analysis (PCA). Even though each of them have different derivations, Wu et al. (2003) showed the equivalence between those methods and how they can all produce the same solution.

In this paper, KLD has been chosen to carry out the POD calculations. Suppose $\mathbf{U}$ is a set of displacement fields where each describes the state of deformation of a body at discrete time steps t_i, $i = 1 \ldots n$, defined over a one-dimensional spatial domain.

$$\mathbf{U} = \left\{\mathbf{u}^1, \mathbf{u}^2, \ldots, \mathbf{u}^n\right\}, \tag{1}$$

where for the total number of time steps it holds $n << m$, and m is the number of displacement degrees of freedom. If the displacement field is approximated by a set of basis vectors, $\widetilde{\mathbf{\Phi}}$, and coefficients, $\boldsymbol{\alpha}$, through