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A parametric study of the mechanical and dispersion properties of cubic lattice structures

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ABSTRACT

A study of the mechanical and dispersion properties of cubic lattice structures have been conducted to assess the viability of designing a multifunctional and lightweight lattice structure with excellent static properties and elastic band gaps for vibration attenuation. In this study, the parameters that characterises the mechanical properties for stiffness and strength to be used as sandwich structure core materials were identified. A parametric study on the geometry of the lattice structures on the static properties was performed in order to determine the optimal geometry for these applications. The trends relating the geometric parameters to the mechanical properties of the lattice topologies were found and discussed. Local resonators were then added to the optimal geometries to create the band gaps that will attenuate vibrations at given frequency ranges. The tuned frequency was set to be 500 Hz in this study. The effects of the geometric parameters on the band gap widths produced by the introduction of the resonators were studied and the trends were found to be similar for all topologies. The results of this study indicate that the addition of local resonators to introduce band gaps is only viable when the stiffness and strength of the lattice without the resonators are sufficiently large, so that the increase in density will not be too significant. Lastly, band gaps around the tuned frequencies were observed for one of the topologies without the resonators. Since these band gaps do not result in an increase in mass, tuning the geometry to move the band gaps to the desired frequency ranges is a preferred strategy over the addition of local resonators.

1. Introduction

Lattice materials are cellular structures composed of periodically repeating unit cells. These materials can be engineered to have several unique properties, such as having high strength to density ratio (Deshpande et al., 2001; Fleck et al., 2010), high stiffness to density ratio (Deshpande et al., 2001; Fleck et al., 2010), low thermal coefficient with high stiffness (Berger et al., 2011; Steeves et al., 2007; 2009), and elastic band gaps, which are regions of frequencies that prevent elastic waves from propagating (Leamy, 2012; Phani et al., 2006; Raghavan and Phani, 2013). Since lattice materials can be designed to have several desirable properties, they have huge potential to be used in multifunctional applications, including ultralight structures, impact absorbers, heat dissipation, vibration control, and many others.

The multi-functionality of lattice materials enables it overcome several issues that cannot be resolved with conventional bulk materials. Among these issues is the conflicting requirements of stiffness and vibration control of structural components. In order to sustain huge loads without large static displacements or compromising structural integrity, structural components generally require high stiffness (Baravelli et al., 2013; Lagoudas et al., 2001). However, structures with high stiffness are also sensitive to vibrations, which may threaten the structural integrity of the system. A possible solution to minimise the trade-off between vibration attenuation and stiffness is to exploit the high stiffness and strength to density ratios present in selected lattice structures (Ashby, 2006; Ashby et al., 2000) and embedding local resonators, as seen in several studies (Baravelli et al., 2013; Baravelli and Ruzzene, 2013; Liu et al., 2012; 2011), to produce band gaps that will attenuate vibrations. With current three-dimensional printing technology, lattice structures with the embedded local resonators can be manufactured, allowing the potential of the multi-functionality of lattice structures to be realised.

Currently, there are a large number of studies that have been conducted on the analysis of the static mechanical (stiffness and strength) (Choi et al., 2010; Deshpande et al., 2001; Dragoni, 2013; Mines, 2008; Ptochos and Labeas, 2012a; 2012b; Vigliotti and Pasini, 2012; Wallach and Gibson, 2001) and dispersion properties

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of lattice materials (Leamy, 2012; Liu et al., 2012; 2014; Phani et al., 2006; Raghavan and Phani, 2013; Xu et al., 2013). However, only a few studies (Baravelli et al., 2013; Baravelli and Ruzzene, 2013; Liu et al., 2011; Scarpa et al., 2013) have investigated these two types of properties in tandem or the design of a stiff structural material with desirable dynamic characteristics. Therefore, this paper aims to contribute to the design of elastic metamaterials with desirable dynamic properties, while having other qualities, like high stiffness, so that it can be implemented in practical situations. This is an important feature of elastic metamaterials as highlighted in Hussein et al. (2014). The parametric study of the effects of the geometry on the static mechanical and dispersion properties will serve as a guide for future designs of multifunctional components and encourage future research in this type of materials.

In this study, the mechanical and dispersion properties of four cubic lattice topologies with internal resonators, similar to the designs seen in Liu et al. (2012), were studied. Firstly, a parametric study on the lattice geometry, which are the lengths and radii of the lattice struts, was performed to determine the combination of geometric parameters that give the optimal mechanical properties. Local resonators consisting of struts with a point mass at one of its ends will then be attached to the lattices with the best mechanical properties to produce the local resonant band gaps. The width and location of the band gaps resulting from the addition of local resonators of different sizes and masses will be investigated. Lastly, the increase in density was also calculated to determine the penalty of adding the resonators and to assess if this strategy is viable.

2. Selected topologies and analysis procedures

2.1. Selected topologies and resonators

Four cubic lattice topologies, which are the simple cubic, body centred cubic (BCC), face centre cubic (FCC), and octet truss structures, to be used as core materials in sandwich structures were studied. The unit cells of these topologies including the local resonators are shown in Fig. 1. Several of the beams in the unit cells were omitted to avoid overestimating the strength and stiffness through duplication. The lattice structures will consist of the *base* lattice structures and the added local resonators, which are beams with a point mass, for example a ball bearing, at the end. This facilitates the manufacturing of the local resonators as materials other than the strut materials can be used for the mass, which can be attached easily via other strategies like adhesives.

There are several reasons for the selection of these topologies. Firstly, the addition of the resonators to these lattices are straightforward and since the resonators do not contribute to the strength or stiffness of the material, the mechanical properties and dispersion properties can be investigated independently. Additionally, cubic lattice structures can be easily manufactured using currently available three-dimensional printing technologies. The natural frequencies of the local resonators, and the location of the band gap, can also be easily tuned without making any changes to the base structure.

Although other methods to create LR band gaps, such as adding masses at the nodes (Liu et al., 2014) or manipulating the connectivity of the struts as discussed in Wang et al. (2015) can be used to introduced band gaps, the key reason that the topologies in Fig. 1 were selected over the other architectures is because the natural frequencies of the resonators can be found easily. This facilitates the selection of the geometric parameters and mass of the resonators to achieve the required tuned frequencies. The calculations for the target frequencies by adding masses at the nodes are less straightforward than the cantilever resonators in the selected designs. Furthermore, the lattice structure studied here is intended to be used as a lightweight structural component, with high stiffness and low mass, suggesting that the struts are likely to be stiff. Since the natural frequency increases with increasing stiffness and reducing mass, the strategy of adding masses at the nodes may lead to large masses being required for a given target frequency. Conversely, the resonators that do not take any load can be designed to have low stiffness and a lower mass can be used to achieve the same natural frequency. This provides more design flexibility. Similarly, the manipulation of the strut connectivity will result in significant changes in the static mechanical properties, which complicates the analysis significantly.

Table 1
Ratios to maximise for beam and plate configurations (Ashby et al., 2000).

Configuration	Strength constrained	Stiffness constrained
Beam	$\dfrac{\sigma_f^{\frac{2}{3}}}{\rho}$	$\dfrac{E^{\frac{1}{2}}}{\rho}$
Plate	$\dfrac{\sigma_f^{\frac{1}{2}}}{\rho}$	$\dfrac{E^{\frac{1}{3}}}{\rho}$

where σ_f is the failure stress, E is the Young's modulus, and ρ is the density.

The ratios characterising the structural performance of core materials in lightweight sandwich panels and beams according to Ashby et al. (2000) are summarised in Table 1. The ratios in the table were obtained using the analysis from Ashby (2005), which will be described briefly. For a beam or panel being loaded as shown in Fig. 2 to be used in lightweight applications, the mass of the beam or panel, m, defined in Eq. (1) must be minimised. For a beam the width, W, and height, H, are equal while a panel is assumed to have a fixed width, W,

$$m = \rho(WHL) \tag{1}$$

where ρ is the density of the material and W, H, and L are the dimensions shown in Fig. 2.

However, as a structural bearing component, the beam or panel must be able to either have a sufficient amount of stiffness (stiffness constrained) to minimise deflection or sufficient strength (strength constrained) to prevent failure. Therefore, the minimum stiffness, S_E and strength, S_σ are defined in Eq. (2) and Eq. (3), respectively. These equations are easily derived based on the Euler–Bernoulli beam theory and although Fig. 2 shows a point load condition, the equations for other loading conditions can be found by varying the variable C. Interested readers can refer to Ashby (2005) for the derivations of the equations and values of C.

$$S_E \le \left(\frac{F}{\delta}\right) = \left(\frac{CWH^3}{12L^3}\right)E \tag{2}$$

$$S_\sigma \le \left(\frac{F}{WH}\right) = \left(\frac{CWH^2}{6L}\right)\sigma_f \tag{3}$$

where S_E and S_σ are the stiffness and strength constraints, F is the force, δ is the deflection, E is the Young's modulus, and σ_f is the failure stress.

According to Eq. (1), the values of H and W for the beam, and H for the panel can be varied freely. The restrictions of the stiffness and strength constraints are imposed by substituting Eqs. (2) and (3) into Eq. (1) to eliminate $H = W$ for the beam, or H for the panel.

$$m = \left(\frac{12S_EL^3}{C}\right)^{\frac{1}{2}} L\left(\frac{\rho}{E^{\frac{1}{2}}}\right) \tag{4}$$

$$m = \left(\frac{6S_\sigma L}{C}\right)^{\frac{2}{3}} L\left(\frac{\rho}{\sigma_f^{\frac{2}{3}}}\right) \tag{5}$$