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Equivalent in-plane elastic properties of irregular honeycombs: An analytical approach

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ABSTRACT

An analytical formulation has been developed in this article for predicting the equivalent elastic properties of irregular honeycombs with spatially random variations in cell angles. Employing unit-cell based approaches, closed-form expressions of equivalent elastic properties of regular honeycombs are available. Closed-form expressions for equivalent elastic properties of irregular honeycombs are very scarce in available literature. In general, direct numerical simulation based methods are prevalent for this case. This paper proposes a novel analytical framework for predicting equivalent in-plane elastic moduli of irregular honeycombs using a representative unit cell element (RUCE) approach. Using this approach, closed-form expressions of equivalent in-plane elastic moduli (longitudinal and transverse Young's modulus, shear modulus, Poisson's ratios) have been derived. The expressions of longitudinal Young's modulus, transverse Young's modulus, and shear modulus are functions of both structural geometry and material properties of irregular honeycombs, while the Poisson's ratios depend only on structural geometry of irregular honeycombs. The elastic moduli obtained for different degree of randomness following the proposed analytical approach are found to have close proximity to direct finite element simulation results.

1. Introduction

Honeycombs have gained considerable attention in recent years as an advanced material due to its capability of meeting high performance requirements in various critically desirable application-specific parameters. These include specific strength and stiffness, electro-mechanical properties, acoustic properties, shock absorption, fatigue strength, corrosion and fire resistance. Such lattice and/or lattice-like structures are present in materials and structures across different length-scales. The use of honeycomb core in several applications of sandwich structures is an important area of research (Yongqiang and Zhiqiang, 2008; Zenkert, 1995). An in-depth analysis of the structural behavior of honeycomb can be useful in emerging research areas such as carbon nano-materials like graphene, as these are generally idealized to have hexagonal periodic structural forms (Liu et al., 2012; Pantano et al., 2004; Scarpa et al., 2009).

Honeycombs are modeled as a continuous solid having an equivalent elastic moduli throughout its domain. This approach eliminates the need of detail finite element modeling of honeycombs in complex structural systems like sandwich structures. To date, extensive amount of research has been carried out to predict the equivalent elastic properties of regular honeycombs consisting of perfectly periodic hexagonal cells (El-Sayed et al., 1979; Gibson and Ashby, 1999; Goswami, 2006; Zhang and Ashby, 1992). Constitutive models for two-dimensional linear as well as non-linear elastic foams have been developed in (Warren and Kraynik, 1987) and (Warren et al., 1989) respectively considering an appropriate representative volume element to analyse periodic foam structure. Elasto-plastic yield limits and failure surfaces for large deformations of transversely crushed honeycombs have been analyzed using theoretical predictions in (Klintworth and Stronge, 1988). Recently numerical investigations of buckling and crushing behavior of expanded honeycomb are found to be carried out by Jang and Kyriakides (2015), while Wilbert et al. (2011) have studied buckling and progressive crushing of laterally loaded honeycombs. Other important research areas concerning the study of different responses related to periodic honeycombs include low velocity impact (Hu and Yu, 2013) and buckling analysis (Lopez Jimenez and Triantafyllidis, 2013) and wave propagation through lattices (Schaeffer and Ruzzene, 2015). There is a substantial amount of literature available on the study of perfectly periodic hexagonal auxetic honeycombs (Critchley et al., 2013; Rossiter et al., 2014; Scarpa et al., 2000). Of late theoretical formulations for equivalent elastic properties of periodic asymmetrical honeycomb have been

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developed in (Chen and Yang, 2011), while the tailorable properties of hierarchical honeycombs, including spiderweb honeycombs have been investigated in (Ajdari et al., 2012; Mousanezhad et al., 2015). Analysis of two dimensional honeycombs dealing with in-plane elastic properties presented in the above survey are commonly based on unit cell approach, which is applicable only for perfectly periodic cellular structures.

A significant limitation of the aforementioned unit cell approach is that it cannot account for the spatial irregularity, which is practically inevitable. Spatial irregularity in honeycomb may occur due to manufacturing uncertainty, structural defects, variation in temperature, pre-stressing and micro-structural variability in honeycombs. To include the effect of irregularity, voronoi honeycombs have been considered in several studies (Li et al., 2005; Zhu et al., 2001; 2006). Dynamic crushing behavior of honeycomb structures with irregularity in cell shapes and cell wall thickness have been investigated in (Li et al., 2007). Triantafyllidis and Schraad (1998) have reported study on failure surface of aluminium honeycombs under general in-plane loading to compare the theoretical results, obtained for the infinite, perfectly periodic honeycomb model and the numerical results, obtained for the finite counterpart with micro-structural imperfections considering uncertainties in manufacturing and fabrication. Jang and Kyriakides (2015); Papka and Kyriakides (1994); 1998) carried out numerical and experimental study of honeycomb buckling and crushing behavior considering geometrical imperfections in the structure such as variation in length of bond line and over or under expanded cells. Though these studies substantially explore the effect of imperfections as pioneering works, a further need is felt to extend these works for spatially random imperfections to develop more realistic model of the uncertainties associated with such irregularities. Stochastic multi-scale analysis for the elastic properties of honeycombs have been presented in more recent studies (Basaruddin et al., 2014). The effect of defects on the behavior of regular as well as voronoi honeycombs (Ajdari et al., 2008), and the effect of manufacturing irregularity on auxetic honeycomb (Liu et al., 2014) have been investigated. In the studies involving voronoi honeycombs, the shape of all irregular cells generated using voronoi diagram may not be necessarily hexagonal, which violates the presumption of hexagonal cell structure in many applications. Published researches that explore the effect of different forms of irregularity on elastic properties and structural responses of honeycombs are based on either experimental investigations or expensive finite element (FE) simulation. Experimental investigations, being very expensive and time consuming, its practically not feasible to capture the effect of random irregularities in honeycomb structure by testing huge number of samples. In finite element approach, a small change in geometry of a single cell may require completely new geometry and meshing of the entire structure. In general this makes the entire process time-consuming and tedious. For quasi-static and dynamic analysis, finite element modeling of the cellular core in a sandwich panel may increase the degree of freedom of the entire structure up to huge extent, making the overall process more complex and prohibitively expensive to simulate. The problem becomes even worse for uncertainty quantification of the responses associated with irregular honeycombs, where the expensive finite element model is needed to be simulated for a large number of samples while using a Monte Carlo based approach (Dey et al., 2015a; 2015b; 2015c; Hurtado and Barbat, 1998). Direct numerical simulation to deal with irregularity in honeycombs may not necessarily provide proper understanding of the underlying physics of the system. An analytical approach could be a simple, insightful, yet an efficient way to obtain effective elastic properties of honeycombs.

This paper develops an analytical framework for predicting equivalent in-plane elastic properties of irregular honeycomb having spatially random variations in cell angle. Geometrical imperfections due to over or under expanded cells have been considered by Papka and Kyriakides (1994). However, random spatial distribution of over or under expanded cells has not been considered yet, which can be a realistic and logical extension of the previous work. As this article proposes closed-form formulae for such irregularities, the responses can be investigated in a more robust but efficient manner. Towards the development of explicit analytical formulae of in-plane elastic moduli for addressing any such form of irregularity in cellular structures, this is the first attempt of its kind to the best of authors' knowledge. closed-form formulae developed here can be a computationally efficient and less-tedious alternative to the expensive finite element modeling and simulation approach for many applications. This article is organized as follows. Derivations of formulae for five in-plane elastic moduli of irregular honeycombs are described in Section 2. Development of finite element model to obtain the in-plane elastic moduli numerically and validation of the finite element code with available literature (Gibson and Ashby, 1999) are discussed in Section 3. Variations of elastic moduli for different degree of random variations in the cell angle and comparison of results between the proposed analytical approach and finite element simulation are detailed in Section 4. Finally, Section 5 summarises the main findings and draws conclusions based on the results obtained in the paper.

2. Elastic properties of irregular honeycombs

The key idea to obtain the effective in-plane elastic moduli of the entire irregular honeycomb structure is that it is considered to be consisted of several representative unit cell elements having different individual elastic moduli. Elastic properties of each representative unit cell element (RUCE) depends on its structural geometry and material properties. The irregularity is accounted implicitly by means of the RUCEs. The RUCE considered in this study for deriving the expressions of different in-plane elastic moduli for an irregular honeycomb structure is shown in Fig. 1(b). The expressions for elastic moduli of a RUCE is derived first and subsequently the expressions for effective in-plane elastic moduli of the entire irregular honeycomb are derived by assembling the individual elastic moduli of these RUCEs using basic principles of mechanics as discussed in the preceding sections. These formulae are applicable for both tensile as well as compressive stresses.

2.1. Longitudinal Young's modulus (E_1)

To derive the expression of longitudinal Young's modulus for a RUCE (E_{1U}), stress σ_1 is applied in direction-1 (refer figure Fig. 1) as shown in Fig. 2. The inclined cell walls having inclination angle α and β do not have any contribution in the analysis, as the stresses applied on them in two opposite directions neutralise each other. The remaining structure except these two inclined cell walls is symmetric. The applied stresses cause the inclined cell walls having inclination angle θ to bend. From the condition of equilibrium, the vertical forces C in the free-body diagram of these cell walls (refer Fig. 2(b)) need to be zero. In the present analysis the cell walls are treated as beams of thickness t, depth b and Young's modulus E_s. l and h are the lengths of inclined cell walls having inclination angle θ and the vertical cell walls respectively. From Fig. 2(b),

$$M = \frac{Pl\sin\theta}{2} \tag{1}$$

where

$$P = \sigma_1(h + l\sin\theta)b \tag{2}$$

From the standard beam theory (Roark and Young, 1976), the deflection of one end compared to the other end of the cell wall