International Journal of Solids and Structures 94–95 (2016) 206–221

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Responses of an anisotropic magnetoelectroelastic and layered half-space to internal forces and dislocations

Y.S. Li[a], E. Pan[b,*]

[a] College of Civil Engineering, Hebei University of Engineering, Handan 056038, PR China
[b] Department of Civil Engineering, The University of Akron, OH 44325-3905, USA

ARTICLE INFO

Article history:
Received 14 January 2016
Revised 21 March 2016
Available online 9 April 2016

Keywords:
Layered system
Magneto-electro-elasticity
Force and dislocation
Propagator matrix method
Stroh formalism

ABSTRACT

Layered multiferroic heterostructures have applications in various mechanical and electronic devices where the coupling among the mechanical, electric and magnetic fields provides unique opportunity for tailoring the device properties. In this paper, we investigate the coupled magnetoelectroelastic (MEE) field in a layered anisotropic MEE half-space induced by the internal force (traction) and dislocation. Utilizing the Fourier transform and propagator matrix method, we derive the induced field quantities in terms of the Stroh formalism. The analytical solution in the corresponding homogeneous MEE half-space is also derived to remove the concentration/singularity in the layered system. Numerical examples are presented in terms of contours for the extended displacement and stress fields in a three-layered MEE half-space made of piezoelectric and piezomagnetic layers induced by an internal force and dislocation applied over a horizontal circle. These results not only show various interesting features, they can also serve as benchmarks for future numerical analysis and perhaps as guidance for design engineers.

1. Introduction

Magneto-electro-elastic (MEE) materials have different phases within them including elastic, electric, and magnetic ones which can interact for optimal responses (Nan et al., 2008). These materials are usually composites made of multi-phase laminae and exhibit magnetoelectric (ME) coupling effect which is absent in the single-phase piezoelectric or piezomagnetic material. In material science, the term multiferroics is used to indicate simultaneous presence of two or three primary ferroic properties (i.e., ferromagnetism, ferroelectricity and ferroelasticity) (Schmid, 1994, 2008).

Multiferroic composites can be epitaxially grown by combing layers of perovskites and spinels. Multiferroic $BaTiO_3$–$CoFe_2O_4$ composite structure was produced using unidirectional solidification from eutectic compositions (Van den Boomgaard et al., 1978). Also, particulate ceramic composite of ferrites and $BaTiO_3$ can be produced by sintering process (Lopatin et al., 1994). Heteroepitaxial strain, precise composition, atomic arrangements, and interface are important factors in thin-film growth of multiferroics (Choi et al., 2004). Progress and prospect including growth, characterization, and understanding of layered multiferroics can be found in reviews, e.g., by Ramesh and Spaldin (2007) and Nan et al. (2008).

In the past decade, various analytical solutions have been derived in order to better understand the MEE composites. These include various point-source Green's functions in two-dimensional and three-dimensional (3D) anisotropic MEE homogeneous and bimaterial solids (Liu et al., 2001; Li 2002; Pan, 2002; Ding et al. 2005). Furthermore, for an MEE half-space, Chen et al. (2010) presented a unified fundamental theory to deal with the contact problem between a rigid punch (or indenter) and the half-space, and Wang et al. (2012) derived the analytical solution for two types of surface loadings over an anisotropic MEE half-space: a uniform and an indentation-type loading. For the MEE bimaterial case, while Chu et al. (2013) derived the analytical solution of the extended traction and dislocation uniformly distributed over a horizontal ellipse, Zhao et al. (2013) derived the analytical solutions of the extended dislocations and tractions generally distributed over a horizontal circular area by virtue of the extended Stroh formalism and Fourier transformation. For the layered MEE half-space, under the assumption of transverse isotropy and by introducing the cylindrical system of vector functions along with the propagating matrix, Chu et al. (2011) presented a semi-analytical solution of a uniform vertical load over a circular area on the surface. While anisotropic and layered composites are very popular with many applications (i.e., Wu et al. 2016a, b), analytical solutions in these systems (layered and general anisotropy) are still missing, which motivates the present research.

* Corresponding author. Tel.: +1 330 972 6739; fax: +1 330 972 6020.
E-mail address: pan2@uakron.edu (E. Pan).

http://dx.doi.org/10.1016/j.ijsolstr.2016.04.004
0020-7683/

In this paper, we present the analytical solution in a layered half-space containing general anisotropic MEE materials due to the extended force (traction) and dislocation applied in a horizontal circle within the half-space. The paper is organized as follows: in Section 2, we briefly present the basic equations associated with the anisotropic MEE material. We then derive the solution in the Fourier-transformed domain in Section 3. In Section 4, we describe the layered half-space problem to be solved in the Fourier-transformed domain. Treatment of the associated singularity/concentration in the layered half-space is discussed in Section 5. Numerical results are presented in Section 6, and conclusions are drawn in Section 7.

2. MEE coupled governing equations

Under static deformation, the governing equations for a linear, anisotropic MEE solid can be summarized in terms of the extended notations below (Pan, 2002).

The equilibrium equations without internal source can be recast into

$$\sigma_{iJ,i} = 0. \tag{1}$$

In this and other equations, a subscript comma denotes the partial differentiation with respect to the coordinate and a repeated lowercase (uppercase) index takes the summation from 1 to 3 (5). Also in Eq. (1), the extended stresses are defined as

$$\sigma_{iJ} = \begin{cases} \sigma_{ij} & (J = j = 1, 2, 3) \\ D_i & (J = 4) \\ B_i & (J = 5) \end{cases} \tag{2}$$

with σ_{ij}, D_i and B_i being, respectively, the stress, electric displacement and magnetic induction.

The coupled constitutive relation can be written as:

$$\sigma_{iJ} = c_{iJKl}\gamma_{Kl}, \tag{3}$$

where c_{iJKl} are the extended material coefficients, defined as

$$c_{iJKl} = \begin{cases} c_{ijkl} & (J, K = j, k = 1, 2, 3) \\ e_{lij} & (J = j = 1, 2, 3;\ K = 4) \\ e_{ikl} & (J = 4;\ K = k = 1, 2, 3) \\ q_{lij} & (J = j = 1, 2, 3;\ K = 5) \\ q_{ikl} & (J = 5;\ K = k = 1, 2, 3) \\ -\alpha_{il} & (J = 4, K = 5 \text{ or } K = 4, J = 5) \\ -\varepsilon_{il} & (J, K = 4) \\ -\mu_{il} & (J, K = 5) \end{cases} \tag{4}$$

where c_{ijlm}, e_{ijk} and q_{ijk} are, respectively, the elastic moduli (N/m^2), piezoelectric (C/m^2), and piezomagnetic (N/(A × m)) coefficients; ε_{ij} and μ_{ij} are, respectively, the dielectric (C^2/(N × m^2)) and magnetic permeability (N × s^2/C^2) coefficients; and α_{ij} are the ME coefficients (C/(m × A) = Wb/(m × V)).

Also in Eq. (3), the extended strains are defined as

$$\gamma_{Ij} = \begin{cases} \gamma_{ij} \equiv (u_{i,j} + u_{j,i})/2 & (I = i = 1, 2, 3) \\ -E_j \equiv \varphi_{,j} & (I = 4) \\ -H_j \equiv \psi_{,j} & (I = 5) \end{cases} \tag{5}$$

with the extended displacements being

$$u_I = \begin{cases} u_i & (I = i = 1, 2, 3) \\ \varphi & (I = 4) \\ \psi & (I = 5) \end{cases} \tag{6}$$

In Eqs. (5) and (6), γ_{ij}, E_i and H_i are, respectively, the elastic strain, electric field, and magnetic field, and u_i, φ, and ψ are, respectively, the elastic displacement, electric potential, and magnetic potential.

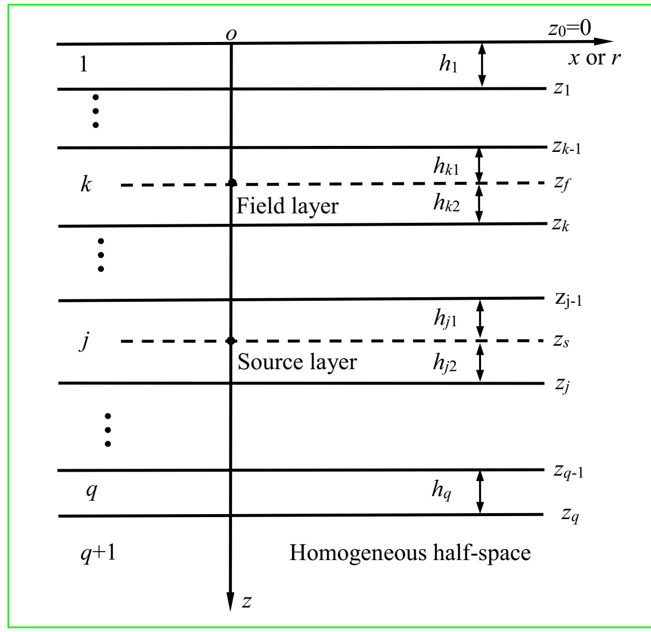

Fig. 1. A layered MEE half-space made of q layer over a homogeneous MEE half-space, subjected to an internal source located in layer j.

Table 1
Material properties of piezoelectric $BaTiO_3$ and magnetostrictive $CoFe_2O_4$ (Chen et al. 2010).

	$BaTiO_3$	$CoFe_2O_4$
c_{11} (GPa)	166	286
c_{12} (GPa)	77	173
c_{13} (GPa)	78	170.5
c_{33} (GPa)	162	269.5
c_{44} (GPa)	43	45.3
e_{31} (Cm^{-2})	−4.4	0
e_{33} (Cm^{-2})	18.6	0
e_{15} (Cm^{-2})	11.6	0
ε_{11} (C^{2}N^{-1}m^{-2})	11.2×10^{-9}	0.08×10^{-9}
ε_{33} (C^{2}N^{-1}m^{-2})	12.6×10^{-9}	0.093×10^{-9}
q_{31} (NA^{-1}m^{-1})	0	580.3
q_{33} (NA^{-1}m^{-1})	0	699.7
q_{15} (NA^{-1}m^{-1})	0	550
μ_{11} (Ns^{2}C^{-2})	5.0×10^{-6}	590×10^{-6}
μ_{33} (Ns^{2}C^{-2})	10.0×10^{-6}	157×10^{-6}

We finally define the extended tractions as

$$t_J = \sigma_{iJ}n_i = \begin{cases} \sigma_{ij}n_i & (J = j = 1, 2, 3) \\ D_i n_i & (J = 4) \\ B_i n_i & (J = 5) \end{cases} \tag{7}$$

where n_i is the normal vector of a prescribed plane.

3. General solutions in Fourier-transformed domain

To solve the problem, we rely on the two-dimensional (2D) Fourier transforms. We first define the 2D Fourier transform as

$$\tilde{f}(\xi_1, \xi_2, x_3) = \iint f(x_1, x_2, x_3)\, e^{\mathrm{i}\xi_\alpha x_\alpha} dx_1 dx_2, \tag{8}$$

where $\mathrm{i} = \sqrt{-1}$ is the imaginary unit; the repeated Greek index α takes the summation from 1 to 2, and ξ_1 and ξ_2 are the Fourier variables.