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Homogenization of a cracked saturated porous medium: Theoretical aspects and numerical implementation

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ABSTRACT

The purpose of this paper is to determine, via a homogenization technique and in the framework of small strains, the macroscopic poroelastic properties of a saturated, deformable, cracked porous medium. The poroelastic matrix is assumed to be homogeneous and the cracks to be connected discontinuities, infilled with a poroelastic material. They are periodically distributed, with the size of the period being small compared to the size of the sample. The considered up-scaling method (based on asymptotic expansions) will provide two uncoupled mechanical and hydraulic problems describing the overall behavior of the material. The degradation of the mechanical properties due to damage is then introduced. Damage depends on cracks' opening, thus making the problem non-linear. A numerical solution of the problem is provided using finite elements. Any stress-strain loading path can be reproduced. The numerical solution of an oedometric test and a biaxial test allows the exploration of the non-linear anisotropic behavior along with the bifurcation phenomenon.

1. Introduction

During the past decades, big efforts have been made to better comprehend heterogeneous materials (Auriault, 1991; Biot, 1941; Chambon et al., 2004; Kanouté et al., 2009; Sánchez-Palencia, 1980). Numerical modeling can be successfully applied to continuum problems but difficulties arise when trying to model a heterogeneous microstructure (Kouznetsova et al., 2001). The difference between the scale of the micro- and macro-structures makes it difficult to determine an appropriate mesh size, leading to a computationally expensive problem if one focuses on the micro-scale, or to an approximate description of the microstructural behavior if one focuses on the macroscale problem (Kouznetsova et al., 2001).

Furthermore, macro-scale constitutive laws, calibrated with experimental results, are often adopted. This approach is however less effective when dealing with complex behaviors (Caillerie, 2009). An alternative is provided by homogenization techniques that allow the inclusion of the micro-scale description within the macroscopic problem. In this latter framework, analytic, e.g. mixture theory (Gray and Hassanizadeh, 1979) or semi-analytic, e.g. Eshelby (1957) procedures have been developed. However, these theories cannot describe the micro–macro-behavior for non-linear constitutive laws or non-regular micro-structure configurations in an accurate manner (see, e.g. Kanouté et al., 2009). Numerical homogenization approaches such as direct micro–macro-techniques (Miehe et al., 1999; Nguyen, 2013; Nitka et al., 2011; Smit et al., 1998) overcome these limitations. These techniques use numerical calculations at the (usually periodic) micro-scale level to provide a constitutive law at the macroscale. Although this approach allows more general multi-scale problems to be taken into account, it is highly computationally expensive.

The asymptotic homogenization theory documented in Arbogast et al. (1990), Bensoussan et al. (2011), Papanicolau et al. (1978), Sánchez-Palencia (1980) permits equivalent properties to be obtained and allows an analytic and a numerical approach to be combined. Based on asymptotic expansions (applied to a parameter e that relates the characteristic lengths of the two, well-separated, scales), the homogenized problem can be solved on a generic micro-structural cell (solved using, e.g. finite elements Auriault, 2011) so that the homogenized macroscopic properties are finally obtained.

The proposed approach is developed herein with the purpose of determining the overall poroelastic properties of a saturated cracked deformable porous medium in the framework of small strains. We consider the deformation and the porous flow of the medium to be governed by Biot's equations of poroelasticity. The cracks are thin enough to be considered as curved lines (surfaces in 3D) and interconnected, forming a periodic-network. From a mechanical viewpoint, a crack is here considered (differently from other approaches, e.g. Pensée et al., 2002) as an infilled

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discontinuity containing a soft poroelastic material that can undergo damage. This corresponds to a weakened elastic zone allowing its two lips to slip and to move apart. The relative motion of the lips induces a change of the porosity of the crack and consequently a change in the fluid flow. Crack propagation is not treated and the opening of cracks is considered to damage the material, thus affecting the transport properties of the medium. This latter point is consistent with the proposed upscaling procedure (based on asymptotic homogenization) that naturally leads to two uncoupled hydraulic and mechanical problems. It is worth noting that the methodology is not adapted to situations where crack propagation matters. In the case of a hydro-mechanical problem, crack propagation induces a sudden change of the stress/strain field that also affects the pressure field (see Pizzocolo et al., 2013; Schrefler et al., 2015). The proposed model is therefore aimed to treat stationary hydraulic cases rather than transitory states.

In other terms, the upscaling method is aimed at obtaining a material constitutive law for an REV of a porous medium characterized by infilled discontinuities. The numerical behavior law can be finally embedded in any multiscale approach (e.g. Kouznetsova et al., 2001) so that real-scale geomechanics or engineering problems can be treated.

The first part of the paper presents the equations governing the coupled hydro-mechanical problem in the porous matrix and the cracks. The asymptotic homogenization is detailed and the final equations describing the macro-scale problem are presented. In the second part of the paper, the homogenized problem is numerically solved first for the linear case. As a further step, damage is introduced, which makes the problem non-linear. The proposed numerical implementation allows to reproduce any stress/strain loading history: two cases are considered, the first using a strain controlled path (i.e. oedometric test) and the second using a mixed stress/strain condition (i.e. biaxial test). A constitutive non-linear material law can then be obtained for any loading history.

Notations.

- The "usual" vectors: positions, normal, tangent, forces, flows, ... are denoted: $\vec{x}$, $\vec{y}$, $\vec{n}$, $\vec{\tau}$, $\vec{u}$, $\vec{v}$, $\vec{T}$, $\vec{q}$, ... $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$ is an orthonormal basis.
- The dot symbol $\cdot$ denotes the simple contraction between two tensors of any order: $\vec{T}\cdot\vec{v}$, $\vec{T}=\sigma\cdot\vec{n}$, ...
- The colon symbol : denotes the double contraction of two second order tensors: $\sigma : \nabla\vec{v}$, $c:\epsilon(\vec{u})$, ...
- The tensor product $\vec{a}\otimes\vec{b}$ denotes the linear application defined by: $\forall\vec{c}$, $\left(\vec{a}\otimes\vec{b}\right)\cdot\vec{c}=\left(\vec{b}\cdot\vec{c}\right)\vec{a}$.
- $\overrightarrow{\text{grad}}f$ denotes the gradient of the scalar function f, $\nabla\vec{u}$ is the gradient of the vector field $\vec{u}$ and $\epsilon(\vec{u})$ denotes the strain tensor associated to the displacement field $\vec{u}$, i.e. the symmetrical part $\nabla\vec{u}^S$ of $\nabla\vec{u}$. The gradients of a field of two space variables $\vec{x}$ and $\vec{y}$ are distinguished by an exponent: $\overrightarrow{\text{grad}}^x f$, $\nabla^y\vec{v}$.
- Whenever the index notation of tensors is used, the Einstein notation for the contraction of tensors is adopted.

2. Description of a saturated cracked deformable porous medium

2.1. Description of the medium and strong form of the equations

Let us consider a cracked deformable and saturated porous medium occupying, in the small strain framework, a domain Ω. For the sake of simplicity the study is carried out in two dimensions; an extension to 3D is straightforward but is not presented in the following for sake of clarity of the notations. However, some hints about the 3D modeling are given.

The porous parts of the medium are separated by cracks which are curves that joint at points (see Fig. 1); Γ denotes the set of all cracks of the medium. To make the writing of the equations of the poroelasticity of the cracks precise, the cracks are (arbitrarily) oriented, let s denote the curvilinear abscissa along a crack and $\vec{\tau}$ its unit vector, assuming the crack is smooth. The unit normal $\vec{n}$ to a crack is the vector obtained by the rotation of angle $+\frac{\pi}{2}$ of the tangent vector $\vec{\tau}$.

The considered porous medium is assumed to be finely periodic. That means, on one hand, that the space distribution of cracks is periodic (see Fig. 1) and, on the other hand, that the size of the period is small with respect to that of the medium. In the asymptotic expansion method of homogenization used in this paper, the ratio of the size of the period to that of the medium is a small parameter intended to go to 0. That means that the periodic cells of the medium are increasingly smaller. The usual way to handle this is to define the cells of the medium as the image of a given cell Y by a homothety of ratio e, e being the small parameter of the asymptotic procedure (see Bensoussan et al., 2011 and Sánchez-Palencia, 1980).

A function defined on Y is said to be Y-periodic if it takes equal values on opposite sides of the cell Y.

Biot's equations of the porous parts. In the porous parts of the medium Ω, the deformation of the medium and the flow of fluid are governed by Biot's equations that read, see (see Biot, 1941; 1955; Auriault, 2005 or Coussy, 2004):

$$\text{div}\ \sigma = 0 \tag{1a}$$

$$\sigma = c:\epsilon(\vec{u}) - p\alpha \tag{1b}$$

$$\kappa = \alpha:\epsilon(\vec{u}) + \beta p \tag{1c}$$

$$\text{div}\ \vec{q} + \dot{\kappa} = 0 \tag{1d}$$

$$\vec{q} = -k\overrightarrow{\text{grad}}p \tag{1e}$$

where η denotes the porosity of the porous matrix. $\vec{u}$ is the displacement field and $\dot{\vec{u}}$ its time derivative, σ is the total Cauchy stress tensor and p is the pore pressure. $\vec{q}=\eta\left(\vec{v}-\dot{\vec{u}}\right)$ is the relative fluid flow, $\vec{v}$ being the velocity of the fluid. c is the fourth order tensor of elastic stiffness, α is the second order tensor of Biot coefficients, β is the Biot modulus and k is the permeability of the medium. κ denotes the variation - due to the displacement $\vec{u}$ - of the porosity, see Coussy (2004), which reads in terms of the porosity of the porous matrix η and of its variation $\delta\eta$ due to the deformation of the medium:

$$\kappa = \delta\eta + \eta\ \text{div}\,\vec{u}$$

Equations on the cracks. The cracks separating the porous parts of the medium are very soft and highly permeable. That means that the lips of the cracks can slide and open and, in order to maintain coherence, that the stress vector $\vec{T}=\sigma\cdot\vec{n}$ is continuous on the cracks. The displacement field $\vec{u}$ is then discontinuous on the cracks and its jump $\vec{u}^+ - \vec{u}^-$ through a crack where $\vec{u}^+$ is the value of $\vec{u}$ on the side toward which $\vec{n}$ points and $\vec{u}^-$ is the value of $\vec{u}$ on the opposite side, is denoted by $[[\vec{u}]]$. The assumption of high permeability means that fluid pressure p is continuous on the cracks but the fluid flow is discontinuous, the jump of the normal flow is $[[\vec{q}]]\cdot\vec{n}$ where $[[\vec{q}]]$ denotes the jump of $\vec{q}$ across the cracks.

According to these assumptions (see Appendix A), the poroelastic behavior of the cracks is modeled by the following equations:

$$\vec{T} = C\cdot[[\vec{u}]] - p\vec{A} \tag{2a}$$

$$\kappa^c = \vec{A}\cdot[[\vec{u}]] + Bp \tag{2b}$$