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Surface cracking in an orthotropic medium subjected to frictional contact

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ABSTRACT

This article presents an analytical method capable of resolving the coupled problem of surface cracking in an orthotropic elastic medium subjected to frictional contact by a rigid flat punch. Reciprocal influences between the surface crack and the flat punch are accounted for by establishing a fully coupled formulation. Governing partial differential equations involving the displacement components are derived in accordance with plane theory of orthotropic elasticity. General solutions corresponding to mode I and II crack problems and contact problem are obtained employing Fourier transformation techniques. These separate solutions are then reconciled; and three coupled singular integral equations are developed by applying crack surface and contact zone conditions. Singular integral equations are solved numerically through an expansion–collocation method in which the primary unknowns are expanded into series in terms of Jacobi polynomials. Comparisons to the results available in the literature for certain special cases do verify the proposed procedures. Further numerical results are presented to be able to demonstrate the influences of material orthotropy, coefficient of friction, and geometric parameters upon the mixed-mode stress intensity factors and the contact stress.

1. Introduction

Quite a large number of engineering materials utilized in technological applications possess an orthotropic macro-structure. Orthotropy is a reduced type of general anisotropy, which stems from the existence of two orthogonal planes of elastic symmetry within a medium. It is encountered not only in conventional composites like fiber reinforced plates and shells but also in newer material systems such as thin films and coatings. Orthotropy in thin films is a result of the processing method. For instance, electron beam physical vapor deposition induces a columnar thin film structure whereas plasma spray technique causes a lamellar type formation. Because of the importance and common usage of orthotropic materials in both conventional and arising fields of technology, there is a vast amount of literature on related mechanics problems, particularly on contact and fracture mechanics.

Solutions in literature on contact mechanics of orthotropic materials depict a clear picture of the influences of material properties and loading parameters on stress distributions. Certain fundamental findings on contact mechanics of anisotropic half-planes are compiled in the book by Kachanov et al. (2003). More recent advances in contact mechanics of orthotropic media include developments pertaining to indentation (Shi et al., 2003), Hertzian contact (Swanson, 2004), frictional moving punch problems (Zhou et al., 2014), and contact with collinear stamps (Dong et al., 2014). Results on fracture mechanics of orthotropic materials are essential to quantify critical and sub-critical crack propagation phenomena. For this reason, stress intensity factors (SIFs) are computed for a variety of crack configurations. Newer results regarding such research work are generated for multiple interacting cracks (Baghestani et al., 2013), an edge cracked orthotropic strip (Matbuly and Nassar, 2003), a dynamically loaded cracked half-plane (Monfared and Ayatollahi, 2012), an inclined crack in an infinite medium (Nobile et al., 2004), and thermally loaded collinear cracks (Zhong et al., 2013).

Certain types of cracking failures in engineering materials require simultaneous consideration of fracture and contact problems. This is especially the case for brittle materials under the effect of severe contact loadings. Primary cracking mechanisms in the vicinity of contact zones in such materials are: Radial cracking due to Vickers indentation (Page and Knight, 1989), Hertzian cracking due to loading by a blunt indenter (Lawn, 1995), and herringbone cracking due to sliding frictional contact (Suresh et al., 1999). Moreover, surfaces subjected to oscillating frictional forces tend to develop fretting fatigue cracks (Nesladek et al., 2012; Hills and Nowell, 2014). Studies on the behavior of cracks located in the

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vicinity of a contact zone are required to understand these failure mechanisms. Analytical research work has been undertaken to examine fracture in isotropic homogeneous and functionally graded materials caused by sliding frictional contact (Hasebe et al., 1989; Hasebe and Qian, 1998; Dag, 2001; Dag and Erdogan, 2002). However, there has been no prior work on such problems in orthotropic materials.

Formulation of the problem of cracking due to sliding contact for orthotropic materials is substantially different from those developed for isotropic homogeneous and functionally graded materials. Constitutive relations of orthotropic materials contain four elastic constants in the case of plane stress, and seven in the case of plane strain. Two elastic constants and an inhomogeneity parameter are needed for FGMs; whereas for the isotropic homogeneous case specification of two elastic constants suffices. Distinct structure of the constitutive law leads to a different set of partial differential equations for orthotropic materials. Application of Fourier transform techniques then results in completely different general solutions, which depend on the elastic constants of orthotropy. As a consequence, the terms and kernels of the singular integral equations are not same as those found for isotropic homogeneous or functionally graded materials. For this reason, all formulation steps need to be reapplied for orthotropic materials; and general solutions and singular integral equations have to be derived from scratch in terms of engineering constants of plane orthotropy.

The present study puts forward an analytical approach capable of solving the coupled problem of cracking due to sliding contact in an *orthotropic* medium. For this purpose, a surface crack in an orthotropic half-plane in sliding frictional contact with a rigid punch is considered. Governing partial differential equations in terms of the displacement components are derived by employing the elements of plane orthotropic elasticity. Crack and contact problems are formulated separately by means of Fourier transform techniques. Primary unknown functions in these formulations are respectively relative crack surface displacement derivatives and contact stress for the crack and contact problems. These two separate formulations are then reconciled and reduced to a system of three coupled singular integral equations. The integral equations are solved numerically by an expansion–collocation technique, in which primary unknowns are expanded into finite series entailing Jacobi polynomials. Numerical analyses are carried out to compute mode I and II stress intensity factors and contact stress as functions of degree of orthotropy, coefficient of friction, and geometric parameters.

2. Formulation

The coupled problem of surface cracking in an orthotropic medium due to sliding contact is illustrated in Fig. 1. An elastic orthotropic half-plane lies in the region $x_1 > 0$ and $-\infty < x_2 < \infty$. x_1- and x_2-axes are the principal axes of orthotropy. The half-plane contains a crack located at $x_2 = 0$; and is in sliding frictional contact with a rigid flat punch. Contact zone extends from $x_2 = b$ to $x_2 = c$. The elastic medium is assumed to be in a state of either plane stress or plane strain. Normal and friction forces transferred by the contact are respectively designated by P and Q, where $Q = \eta P$, η being the coefficient of friction.

The formulation is based on the constitutive relations of plane orthotropic elasticity, which are expressed as:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{bmatrix} = \begin{bmatrix}C_{11} & C_{12} & 0\\ C_{12} & C_{22} & 0\\ 0 & 0 & C_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\end{bmatrix}, \tag{1}$$

where elements of the stiffness matrix are given by

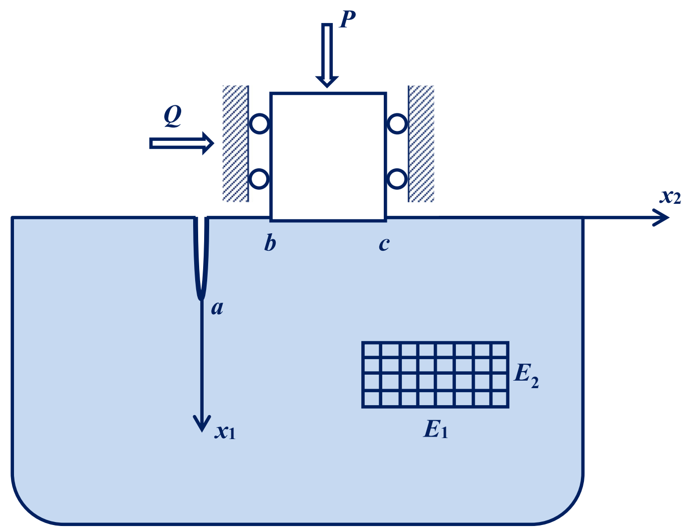

Fig. 1. Geometry of the coupled problem.

$$C_{11} = \begin{cases} \dfrac{E_1}{1-\nu_{12}\nu_{21}}, & \text{for plane stress,}\\ \dfrac{E_1(1-\nu_{23}\nu_{32})}{1-\nu_{12}\nu_{21}-\nu_{13}\nu_{31}-\nu_{23}\nu_{32}-2\nu_{12}\nu_{23}\nu_{31}}, & \text{for plane strain,}\end{cases} \tag{2a}$$

$$C_{12} = \begin{cases} \dfrac{E_2\nu_{12}}{1-\nu_{12}\nu_{21}}, & \text{for plane stress,}\\ \dfrac{E_2(\nu_{12}+\nu_{13}\nu_{32})}{1-\nu_{12}\nu_{21}-\nu_{13}\nu_{31}-\nu_{23}\nu_{32}-2\nu_{12}\nu_{23}\nu_{31}}, & \text{for plane strain,}\end{cases} \tag{2b}$$

$$C_{22} = \begin{cases} \dfrac{E_2}{1-\nu_{12}\nu_{21}}, & \text{for plane stress,}\\ \dfrac{E_2(1-\nu_{13}\nu_{31})}{1-\nu_{12}\nu_{21}-\nu_{13}\nu_{31}-\nu_{23}\nu_{32}-2\nu_{12}\nu_{23}\nu_{31}}, & \text{for plane strain,}\end{cases} \tag{2c}$$

$$C_{66} = 2\,\mu_{12}, \text{ for both plane stress and strain.} \tag{2d}$$

For an orthotropic material, following inequalities are deduced by considering the fact that strain energy density function is positive definite (Agarwal and Broutman, 1990):

$$1-\nu_{12}\nu_{21} > 0,\; 1-\nu_{13}\nu_{31} > 0,\; 1-\nu_{23}\nu_{32} > 0, \tag{3a}$$

$$1-\nu_{12}\nu_{21}-\nu_{13}\nu_{31}-\nu_{23}\nu_{32}-2\nu_{12}\nu_{23}\nu_{31} > 0. \tag{3b}$$

Governing partial differential equations are derived by using the constitutive relations in conjunction with equilibrium equations and kinematic relations; and expressed as given below:

$$d_{11}\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2} + (1+d_{12})\frac{\partial^2 u_2}{\partial x_1 \partial x_2} = 0, \tag{4a}$$

$$\frac{\partial^2 u_2}{\partial x_1^2} + d_{22}\frac{\partial^2 u_2}{\partial x_2^2} + (1+d_{12})\frac{\partial^2 u_1}{\partial x_1 \partial x_2} = 0, \tag{4b}$$

$$d_{11} = \frac{2C_{11}}{C_{66}}, \quad d_{12} = \frac{2C_{12}}{C_{66}}, \quad d_{22} = \frac{2C_{22}}{C_{66}}. \tag{4c}$$

u_1 and u_2 here are the scalar components of the displacement vector in x_1- and x_2-directions, respectively. The solution has to satisfy