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Transient response of a Mode III interface crack between piezoelectric layer and functionally graded orthotropic layer

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ABSTRACT

In this study, transient response analysis of a Mode III interface crack between a piezoelectric layer and a functionally graded orthotropic material (FGOM) layer is conducted using integral transform techniques. Material properties of the FGOM layer change continuously along the layer thickness and two layers are connected weak-discontinuously. Laplace and Fourier transforms are applied to solve the problem, and the problem is then expressed by a Fredholm integral equation of the second kind. It is found that the followings are beneficial to impede transient fracture of the interface crack between piezoelectric layer and FGOM layer: (a) an increase of shear moduli of the FGOM from the interface to the lower free surface; (b) an increase of the gradient of shear moduli, a decrease of ratio of shear moduli, the electric boundary condition EBC I of the piezoelectric layer in the case of increase of shear moduli of the FGOM from the interface to the lower free surface; (c) an increase of thickness of the layers.

1. Introduction

Ever since piezoelectric materials having an electro-mechanical coupling effect were discovered, they have been widely used in various engineering fields. In particular, piezoelectric actuators and sensors for smart structures and structural health monitoring are a popular research area in the aerospace engineering field. Bonding of piezoelectric material to different type of materials such as metal and composite materials, is typical. However, high stress occurs at the bonding surface and the bonding structures often fail.

Recently, attention has been paid to functionally graded materials (FGMs) which can reduce the high stress at the interface as well as be applied in high temperature environments. FGMs have inhomogeneous characteristics because their material properties change gradually. Because of the nature of the techniques used in processing, graded materials are seldom isotropic. Thus, in studying the mechanics of many of the graded materials, an appropriate model would be an inhomogeneous orthotropic elastic continuum (Ozturk and Erdogan, 1997).

The fracture behavior at the interface between piezoelectric materials and the functionally graded orthotropic materials (FGOM) is important for practical usage. Cracks in their interface may experience complex behavior because of their piezoelectric effect, inhomogeneity, and orthotropy.

Dag et al. (2004) studied interface crack problem between an FGOM coating and a homogeneous orthotropic material (HOM) substrate using analytical and finite element techniques. Anti-plane transient fracture analysis of the FGMs with weak/infinitesimal interface was conducted by Li et al. (2006b). Zhou et al. (2010) analyzed a partially insulated interface crack between an FGOM coating and an HOM substrate under heat flux supply. Ding et al. (2014) investigated interface crack behavior for an HOM strip sandwiched between two different FGOMs subjected to thermal and mechanical loading.

Narita and Shindo (1999) studied interface crack in bonded layers of piezoelectric and HOM strips under antiplane shear. Their study showed that the effect of electroelastic interactions on the stress intensity factor and the energy release rate can be highly significant. An anti-plane moving interface crack between a piezoelectric and two HOM layers was analyzed by Lee et al. (2002). This group reported that dynamic stress intensity factor (DSIF) depends on the ratio of stiffness, thickness, the crack length, and the magnitude and direction of electrical loads as well as crack speed. Kwon and Meguid (2002) analyzed a central crack normal to the interface between a rectangular piezoelectric ceramic and two rectangular HOMs under electrical and mechanical loading. The results revealed the field intensity factors and energy release rate are considerably affected by the electrical crack boundary conditions. The Mode-III problem of an interface crack between two dissimilar homogeneous piezoelectric layers under mechanical and electrical impacts was analyzed by means of integral transform method in Gu et al. (2002). Meguid and Zhao (2002) studied the interface

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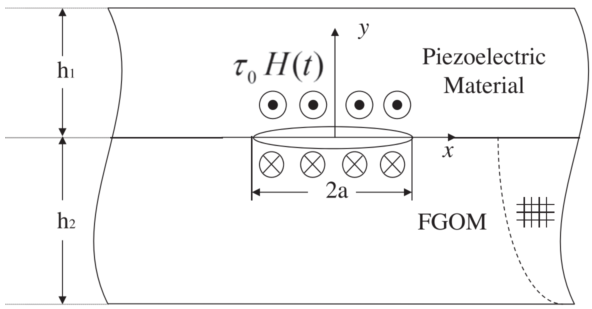

Fig. 1. A Mode III interface crack between piezoelectric layer and functionally graded orthotropic material layer: geometry and loading.

crack problem of bonded piezoelectric and elastic half-space under transient electromechanical loads. Kwon and Lee (2003) examined the steady state dynamic behavior of an eccentric crack moving at constant velocity in a piezoelectric ceramic layer bonded between two HOM layers. DSIF, dynamic energy release rate (DERR) and the crack sliding displacement are presented to show the effects of the crack propagation speed, the crack length, and the electromechanical coupling coefficient. Feng et al. (2011) investigated the problem of multiple cracks on the interface between a piezoelectric layer and an HOM substrate. They suggested the optimal stiffness ratio of the HOM substrate in order to prevent interfacial fracture, which is significant for the design and assessment of smart structures. Problem of an anti-plane crack in the interface of functionally graded piezoelectric and homogeneous piezoelectric materials was analyzed by Dai and Chong (2014). Bayat et al. (2015) presented an analytical model for the analysis of an HOM strip with piezoelectric coating weakened by multiple defects. The analysis showed the stress intensity factor and hoop stress are dependent on the imperfect bonding coefficient, the defect geometry, and the material properties of the HOM substrate. The fracture problem for a medium composed of a cracked piezoelectric strip with FGOM coating was studied by Bagheri et al. (2015).

Despite all of these existing studies, a solution for the transient response of an interface crack between piezoelectric layer and FGOM layer has not been presented to date.

In this study, a transient response analysis of a Mode III interface crack between piezoelectric layer and FGOM layer is conducted. The material properties of the FGOM layer change continuously along the thickness. Weak-discontinuous interface condition is adopted (Li et al., 2006a). Applying Laplace and Fourier transforms makes the problem a dual integral equation. Then the dual integral equation is transformed into a Fredholm integral equation of the second kind. Numerical analyses on the DSIF are carried out to show the effect of piezoelectricity of the piezoelectric layer, orthotropy and inhomogeneity of FGOM layer, and thickness of layers.

2. Problem statement and methods of solution

We consider an interface crack between the homogeneous piezoelectric layer and FGOM layer subjected to anti-plane shear Heaviside step impact loading, as shown in Fig. 1. It is assumed that mechanical loading is applied only. Cartesian coordinates (x, y, z) are fixed to the center of the crack. The piezoelectric layer poled with z-axis and occupies the region, $-\infty < x < \infty,\ 0 \le y \le h_1$. And the FGOM layer occupies the region, $-\infty < x < \infty,\ -h_2 \le y \le 0$. Their thicknesses in the z-direction are infinite in order to make a state of anti-plane shear. The crack is located along the interface line ($-a \le x \le a,\ y = 0$). Only the right-hand half layers are considered due to the symmetry in geometry and loading.

We assume that shear moduli and density of the FGOM layer change continuously along the thickness and are expressed as follows (Ozturk and Erdogan, 1997):

$$c_{44} = c_{440}\exp(\beta y) \tag{1}$$

$$c_{55} = c_{550}\exp(\beta y) \tag{2}$$

$$\rho = \rho_0\exp(\beta y) \tag{3}$$

where c_{44}, c_{55}, and ρ are shear moduli and material density of the FGOM, respectively. c_{440}, c_{550}, and ρ_0 are shear moduli and material density at the interface, respectively, β is the non-homogeneous material constant. It is assumed that the shear modulus (c_{440}) and material density (ρ_0) of the piezoelectric layer are the same as those of the interface line.

The boundary value problem is simplified if the out-of-plane displacement and the in-plane electric fields are considered only such that:

$$u_x^P = u_y^P = 0, \quad u_z^P = w^P(x, y, t) \tag{4}$$

$$E_x^P = E_x^P(x, y, t), \quad E_y^P = E_y^P(x, y, t), \quad E_z^P = 0 \tag{5}$$

$$u_x^F = u_y^F = 0, \quad u_z^F = w^F(x, y, t) \tag{6}$$

where u_k^P and u_k^F ($k = x, y, z$) are the displacements and E_k^P is the electric fields. Superscript P and F denote the piezoelectric layer and FGOM layer, respectively.

In this case, the constitutive equation becomes:

$$\sigma_{zx}^F(x, y, t) = c_{55}w_{,x}^F, \quad \sigma_{zy}^F(x, y, t) = c_{44}w_{,y}^F \tag{7}$$

$$\sigma_{zj}^P(x, y, t) = c_{440}w_{,j}^P + e_{150}\varphi_{,j}^P \tag{8}$$

$$D_j^P(x, y, t) = e_{150}w_{,j}^P - d_{110}\varphi_{,j}^P \tag{9}$$

$$E_j^P(x, y, t) = -\varphi_{,j}^P \tag{10}$$

where σ_{zj} and $D_j (j = x, y)$ are the stress components and electric displacements, respectively. φ, e_{150} and d_{110} are the electric potential, the piezoelectric constant, and the dielectric permittivity measured at a constant strain, respectively.

The dynamic anti-plane governing equations for FGOM and piezoelectric material become expressed as:

$$c_{550}\frac{\partial^2 w^F}{\partial x^2} + c_{440}\beta\frac{\partial^2 w^F}{\partial y^2} + c_{440}\frac{\partial w^F}{\partial y} = \rho_0\frac{\partial^2 w^F}{\partial t^2} \tag{11}$$

$$c_{440}\nabla^2 w^P + e_{150}\nabla^2\varphi^P = \rho_0\frac{\partial^2 w^P}{\partial t^2} \tag{12}$$

$$e_{150}\nabla^2 w^P - d_{110}\nabla^2\varphi^P = 0 \tag{13}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

A new function ψ^P is introduced as follows (Li and Mataga, 1996):

$$\psi^P = \varphi^P - \frac{e_{150}}{d_{110}}w^P \tag{14}$$

By substituting Eq. (14) for Eqs. (12) and (13), the dynamic governing equations are transformed into the following equations:

$$\gamma^2\frac{\partial^2 w^F}{\partial x^2} + \frac{\partial^2 w^F}{\partial y^2} + \beta\frac{\partial w^F}{\partial y} = \frac{1}{c_2^{F2}}\frac{\partial^2 w^F}{\partial t^2} \tag{15}$$