International Journal of Solids and Structures 90 (2016) 228–235

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Finite bending solutions for layered gel beams

P. Nardinocchi [a,*], E. Puntel [b]

[a] Dipartimento di Ingegneria Strutturale e Geotecnica, Sapienza Università di Roma, via Eudossiana 18, Roma I-00184, Italy
[b] Dipartimento Politecnico di Ingegneria e Architettura, Università di Udine, via del Cotonificio 114, Udine I-33100, Italy

ARTICLE INFO

Article history:
Received 28 July 2015
Revised 28 December 2015
Available online 24 February 2016

Keywords:
Morphing beams
Swelling–induced bending
Shape control

ABSTRACT

We investigate the swelling–induced bending of a gel bilayer beam with homogeneous beam–like components, when the beam is embedded in a solvent bath of assigned chemical potential. We set the problem within the limits of finite elasticity with large distortions, with these latter determined via chemical equilibrium, by the elastic modulus of the gel, the chemical potential of the bath and the solvent–gel interaction parameter. We borrow our method of solution from finite bending of soft and incompressible beams, and get a non dimensional nonlinear equation governing the solution of the problem, whose solution is validated through a campaign of numerical experiments based on a complete stress–diffusion model.

1. Introduction

Soft active materials have been largely employed to realize systems in which deformations and displacements are triggered through a wide range of external stimuli such as electric field, temperature, and solvent absorption (Fang et al., 2008; Kim et al., 2012; Nardinocchi and Pezzulla, 2013; Pandey and Holmes, 2013; Teresi and Varano, 2013). In particular, in gel-based actuators the shape and the size of the system are related to the spatial distribution of solvent inside the gel and to the magnitude of the solvent uptake. It was recently demonstrated that gel systems which present non homogeneous elastic modulus (hence, swelling ratio), can generate distinguished deformation patterns under free–swelling conditions. As an example, two-dimensional gel sheets can be morphed into curved surfaces whose characteristics strictly depend on the characteristics of the non homogeneity (Byun et al., 2013; Kim et al., 2012; Wu et al., 2013). Likewise, slender gel beams with elastic modulus varying along the thickness can bend and self-fold; in particular, this last property is attained by forming layered gel systems, with the layers having significantly different expansion ratios and elastic moduli. In a huge bending was induced, appropriately tuning the difference in the geometrical and elastic characteristics of the two layers; therein it was also shown that, under distinguished conditions, gel tubes can be formed by self-folding of bilayered gel systems. The interest in these system is large, as the resulting gel tubes can be proposed for both driving a controlled release of molecules, and realizing morphing slender structures as actuators. The finite bending of a bilayer beam in response to temperature was also discussed in (Morimoto and Ashida, 2015), following the strategy set by Rivlin (1949) and, successively, by Bigoni (2012), and considering the decomposition of the full deformation gradient into a homogeneous swelling component and a pure bending component, as proposed in (Lucantonio et al., 2014a). In particular, the bilayer beam was composed by one swellable (gel) and one not swellable (elastomer) layer; thus, the swelling component of the deformation gradient only interested the gel layer and, due to the presence of the elastomer layer, only an one-dimensional swelling was considered, along the beam's thickness direction.

We concentrate our investigations on layered gel beams, with beam-like homogeneous components. When embedded in a solvent bath, each homogeneous part would swell homogeneously, if it were free from the rest of the system; if the temperature and the chemical potential of the solvent bath were fixed, each part would have a uniform equilibrium swelling ratio depending only on its own shear modulus. When the assembly of two homogeneous beams is considered, because of the possibly different shear moduli and, consequently, different equilibrium swelling ratios, a non-uniform equilibrium swelling is expected which results in a global bending of the system, eventually self–folding. As it is well known, within the context of continuum mechanics, swelling-induced deformation processes can be studied through the so-called stress-diffusion model (Chester and Anand, 2010; Hong et al., 2008; Lucantonio et al., 2013), which was successfully used by one of the Authors to describe bending of bilayered beams

* Corresponding author. Tel.: +39 0644585242; fax: +39 064884852.
E-mail addresses: paola.nardinocchi@uniroma1.it (P. Nardinocchi), eric.puntel@uniud.it (E. Puntel).

http://dx.doi.org/10.1016/j.ijsolstr.2016.02.026
0020-7683/

(Lucantonio et al., 2014a), twisting of bilayered anisotropic sheets (Nardinocchi et al., 2015), and buckling dynamics of a solvent-stimulated stretched elastomeric sheet (Lucantonio et al., 2014b). In (Lucantonio et al., 2014a), it was also shown that it is possible to follow simplified approaches that allow effective modeling of swelling-driven large deformations in non-homogeneous thin structures, based on an uncoupled approach which can be set within the theory of finite elasticity with distortions and allows to establish predictive structure-function relationships for bending of bilayered gel composites. Therein, that approach was followed within the context of a one–dimensional beam theory to determine stretching and bending of the bilayered beam.

Here we use the same kind of approach within the context of the three-dimensional theory of nonlinear elasticity, and look for plane-strain finite bending solutions, presenting and discussing a method borrowed from finite bending of soft and incompressible beams (Bigoni, 2012; Rivlin, 1949). Differently from previous works, the end rotations of the beam are unknown and not given but it has nevertheless been possible to obtain a single, non-dimensional nonlinear equation governing the solution of the problem. This has granted a better discussion and comprehension of the properties of the solution. In subsequent works it may be also possible to use said equation for the determination of closed form solutions under some restrictions on the values of the input parameters. Our results in terms of beam stretching and curvature are in good agreement with the numerical solutions of the stress-diffusion problem of a bilayered gel beam discussed in (Lucantonio et al., 2014a), and also represent an extension of previous results derived with reference to bilayer beams with swellable and not swellable layers. Moreover, our solution of plane strain bending is especially interesting as it deliver an explicit solution for the problem and can be used for incremental bifurcation analysis (see e.g. Dryburgh and Ogden, 1999) to characterize surface instabilities which can start during swelling, when the top layer is much thinner and much softer than the bottom layer (as numerically shown in (Lucantonio et al., 2014a)).

The plan of our paper is the following. In Section 2 the stress-diffusion theory of gels is shortly reviewed and the equilibrium solutions corresponding to a gel body swelling in a solvent bath are presented. In Section 3, the geometry of the problem is first defined and then the notion of elastic distortion is introduced, together with the key elements of the three–dimensional nonlinear theory of elasticity with large distortions; the difference between the elastic distortion and the usual growth distortion is highlighted. Section 4 is devoted to the solution of a preparatory problem concerning the finite bending of a monolayer beam superposed to an isotropic finite growth, inducing only change of volume; in the preparatory problem, bending and swelling (volume change) are two uncoupled processes controlled by two independent parameters, and the problem is set within the context of the plane strain assumption. In Section 5 the finite bending superposed to swelling of a bilayered beam, with homogeneous beam–like parts subject to differential swelling is investigated, and the solution given in terms of both beam stretch and curvature. Finally Section 6 discusses the results, analyzes the response for different values of the input parameters, and establishes a comparison with numerical solutions of the problem coming out from the fully three–dimensional nonlinear stress-diffusion model presented in (Lucantonio et al., 2014a; 2013).

2. Background

Swelling-induced deformation processes can be fully described within a stress-diffusion continuum theory based on the balance equations for forces and solvent, on the thermodynamics inequalities restricting the class of admissible constitutive prescriptions, and on the choice of a free energy density which accounts for both the elastic and mixing contributions. Following (Lucantonio et al., 2013), we identify the gel body at the dry state with its reference configuration $\mathcal{B}_d$, a region of the three–dimensional Euclidean space $\mathcal{E}$ with boundary $\partial\mathcal{B}_d$ of unit normal $\mathbf{m}$. The balance equations of forces and solvent at equilibrium are

$$\operatorname{div}\mathbf{S}=\mathbf{0} \quad \text{and} \quad \operatorname{div}\mathbf{h}=0, \quad \text{on} \quad \mathcal{B}_d, \tag{1}$$

$$\mathbf{S}\,\mathbf{m}=\mathbf{t} \quad \text{and} \quad \mu=\mu_{ext}, \quad \text{on} \quad \partial\mathcal{B}_d, \tag{2}$$

with $\mathbf{S}$, $\mathbf{h}$, and μ as the reference stress, the reference solvent flux, and the chemical potential of the solvent within the body respectively; $\mathbf{t}$ as the boundary traction and μ_{ext} the chemical potential of the solvent bath. Let us note that $\mathbf{t}$ and μ_{ext} are the external agents determining swelling–driven deformation processes. State variables of the problem are the displacement field $\mathbf{u}$ from the reference configuration $\mathcal{B}_d$ and the solvent concentration c per unit reference volume. The constitutive equations for the stress $\mathbf{S}$, the chemical potential μ, and the solvent flux $\mathbf{h}$ come from standard thermodynamical requirements, once fixed the free energy in the form of the Flory–Rehner free energy (Flory and Rehner, 1943a; 1943b), as:

$$\mathbf{S}=\hat{\mathbf{S}}(\mathbf{F})-p\,\mathbf{F}^{\star}=G\mathbf{F}-p\,\mathbf{F}^{\star}, \tag{3}$$

$$\begin{aligned}\mu &= \hat{\mu}(c)+p\,\Omega=\frac{\mathcal{R}\,T}{\Omega}\,h'(c)+p\,\Omega,\\ h'(c) &= \Omega\left(ln\frac{J-1}{J}+\frac{1}{J}+\frac{\chi}{J^2}\right),\end{aligned} \tag{4}$$

$$\mathbf{h}=\hat{\mathbf{h}}(\mathbf{F},p,c)=-\frac{D\,c}{RT}\mathbf{C}^{-1}\,\nabla(\hat{\mu}(c)+p\,\Omega), \quad \mathbf{C}=\mathbf{F}^{T}\mathbf{F}, \tag{5}$$

where

$$\mathbf{F}=\mathbf{I}+\nabla\mathbf{u}, \quad J=\det\mathbf{F}, \quad \mathbf{F}^{\star}=J\,\mathbf{F}^{-T}. \tag{6}$$

Moreover, $G=N\kappa_B T$ ($[G]=\mathrm{Jm}[G]=Jm^{-3}$) is the shear modulus, N the number of polymeric chains per unit dry volume, T ($[T]=\mathrm{K}$) the temperature, Ω ($[\Omega]=\mathrm{m^3mol^{-1}}$) the solvent molar volume, $\mathcal{R}$ ($[\mathcal{R}]=\mathrm{JK^{-1}mol^{-1}}$) the universal gas constant, χ the dimensionless measure of the enthalpy of mixing and D ($[\mathrm{D}]=\mathrm{m^2/s}$) the diffusivity.The field p represents the osmotic pressure within the polymer, when the external pressure is assumed to be zero, and constitutively couples the stress $\mathbf{S}$ and the chemical potential μ, due to the constraint $J=1+\Omega\,c$ which implies that both the volume of the solid and liquid component of the gel do not change, and that the change in volume of the system is only due to the variation in fluid mass content.

The problem (1)–(5) admits a distinguished solution, in the form of a free-swelling solution: the polymer is assumed to be immersed in a solvent bath of chemical potential μ_{ext} under load-free and constraint-free conditions and it can freely swell or shrink. The steady solution $(\mathbf{F}_o, c_o)$ is assumed to satisfy the balance of forces and the local thermodynamic equilibrium:

$$\mathbf{S}(X)=0 \quad \text{and} \quad \mu(X)=\mu_{\mathrm{ext}}, \ \forall X\in\mathcal{B}_d\,; \tag{7}$$

the difference between $\mu(X)$ and μ_{ext} determines the driving force of the swelling (shrinking) process and μ_{ext} is the control parameter of the deformative process. For homogeneous materials, it holds

$$\mathbf{F}_o=J_o^{1/3}\mathbf{I} \quad \text{and} \quad c_o=\frac{J_o-1}{\Omega}, \tag{8}$$

with

$$\frac{G\,\Omega}{J_o^{1/3}}+\mathcal{R}\,T\left(\log\left(\frac{J_o-1}{J_o}\right)+\frac{1}{J_o}+\frac{\chi(T)}{J_o^2}\right)=\mu_{\mathrm{ext}}, \tag{9}$$

and $J_o^{1/3}$ the free-swelling equilibrium ratio. Once fixed the temperature T and the external chemical potential μ_{ext}, the swelling