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Axisymmetric response of a bi-material full-space reinforced by an interfacial thin film

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ABSTRACT

Analytical treatment of a linear elastic isotropic bi-material full-space reinforced by an interfacial thin film under axisymmetric normal loading is addressed. The thin film is modeled as an extensible membrane perfectly bonded to the half-spaces. By virtue of Love's potential function and Hankel integral transform, elastic fields of the system are explicitly written in the form of semi-infinite line integrals. The analytical results are verified by the special cases corresponding to the surface stiffened half-space and classical bi-material problem. The limiting cases of reinforced homogeneous full-space and inextensible membrane are presented and discussed. The proposed formulation is also applicable for studying reinforced auxetic materials with negative Poisson's ratio. The surface/interface effect on the elastic responses of two perfectly bonded half-spaces is also simulated by assigning equivalent surface elastic constant to the membrane stiffness. Effects of thin film stiffness, material properties, loading depth, and surface/interface effect are studied by some numerical examples.

1. Introduction

Wide application of composite materials in recent decades has attracted many researchers to study the elastic responses of bi-material full-spaces as an idealized model of two joined media with different material properties. The bi-material full-space model is also used in geomechanics for studying the behavior of soil strata under buried loading from piles or anchorages.

Rongved (1955) presented a general solution for an isotropic bi-material full-space based on Papkovich functions. Elastic fields in joined half-spaces due to nuclei of strain was studied by Yu and Rath (2000). With the aid of the method of displacement potentials and integral transforms, Guzina and Pak (1999) presented a complete solution to the problem of two joined half-spaces with different material properties subjected to the action of an arbitrary loading. Wang et al. (2016) presented analytical solutions for elastic fields caused by eigenstrains in two joined and perfectly bonded half-spaces. Pan and Yuan (2000), Pan (2007), Khojasteh et al. (2008), and Liew et al. (2001) studied the effects of anisotropy on the behavior of bi-material full-spaces under various types of loading.

Thin films are often used for improving the mechanical characteristics of a medium by utilizing them as buried reinforcing layers or surface coating layers. In geomechanics, geotextiles, geomembranes, and geogrids are employed for reinforcing soil strata. Moreover, material properties of a thin layer on the surface of a half-space can be significantly changed due to corrosion, polishing, and chemical reactions. In order to consider the behavior of bonding material in composites, the bonding layer is often modeled by a thin film.

Since the thickness of thin films is vanishingly small, its disturbing effects in bi-material problems is usually taken into account by proposing a set of interfacial boundary conditions (transmission conditions) between the joined media. There are various types of transmission (interface) conditions in the literature for modeling the behavior of thin films such as inextensible type (Selvadurai, 2009; Shodja et al., 2014), spring type (Benveniste and Miloh, 2001), thin plate type (Eskandari and Ahmadi, 2012; Selvadurai, 2014), membrane type (Ahmadi and Eskandari, 2014a; Argatov and Sabina, 2012; Rahman and Newaz, 1997), etc. The reader is referred to see the extensive list of references cited in Benveniste and Miloh (2001) for more transmission conditions. For a general case of curved isotropic thin film between two isotropic materials, Benveniste and Miloh (2001) considered seven distinct

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interface conditions depending on the stiffness of the thin film with respect to its adjacent media. Later, Benveniste (2013) proposed a set of transmission conditions to model the thin films with variable elastic moduli for plane-strain problems. The more complicated transmission conditions considering nonlinear material behavior of thin film, slip conditions at the interface, etc. requires numerical analysis such as FEM (Mishuris et al., 2006; 2005). Recently, Sonato et al. (2015) introduced general transmission conditions for thin elasto-plastic pressure-dependent interphase between dissimilar materials. Their model is generally nonlinear and requires FEM analysis.

Rahman and Newaz (1997) studied the problem of a transversely isotropic half-space coated with a thin film under a normal ring load. Assuming an extremely flexible behavior for the thin film and averaging stress over the thickness, they proposed a simple transmission condition for axisymmetric problems in terms of thin film in-plane stiffness. In fact, if the stiffness parameter approaches to infinity, the inextensible model is resembled. This simple transmission condition has been frequently used in the literature by several researchers, for instance, see Ahmadi and Eskandari (2014a) and Argatov and Sabina (2012). Recently, following a similar approach, Argatov and Mishuris (2015) studied the problem of a rigid base finite layer coated with an elastic membrane. As another application of the extensible membrane model, Ahmadi and Eskandari (2014a) showed that the behavior of elastic medium with surface/interface effects based on Gurtin's approach (Gurtin and Murdoch, 1975; Gurtin et al., 1998) can be predicted by assigning the equivalent surface elastic constant to the membrane stiffness. To the best of the authors' knowledge, for analytical treatment of elastic medium reinforced by thin films, the elastic membrane model is the most appropriate one available in the literature.

Several researchers considered the elastic responses of bi-materials with imperfect or reinforced interface. Selvadurai (1994) and Gladwell (1999) solved the mixed-boundary-value problem of a bi-material with a rigid disk inclusion on the interface. The reader is referred to see the extensive list of references cited in Ahmadi and Eskandari (2014b). Recently, Selvadurai (2014) studied a bi-material full-space reinforced by a thin plate at the interface with either smooth or adhesive boundary conditions. Utilizing FEM analysis, Mishuris and Öchsner (2007) modeled a thin elasto-plastic interphase between two different materials in plane strain case. The analytical treatments of cracks at the interface of two joined half-spaces were also addressed by Suo and Hutchinson (1990), Qu and Bassani (1993), and Li et al. (2015).

In this paper, the problem of a linear elastic isotropic bi-material full-space reinforced by an interfacial thin film under axisymmetric normal loading is considered. The thin film is modeled as an elastic membrane based on the transmission conditions presented by Rahman and Newaz (1997). By employing Love's potential function, the governing equilibrium equations become decoupled. With the aid of Hankel transform, all elastic fields of the system are explicitly written in the form of semi-infinite line integrals. For verification purposes, the special cases corresponding to the surface stiffened half-space and classical bi-material problem are recovered from the obtained solution. The limiting cases of reinforced homogeneous full-space and inextensible membrane are also presented and discussed. The surface/interface effect on the elastic responses of two perfectly bonded half-spaces is also simulated by assigning equivalent surface elastic constant to the membrane stiffness. In some plots the effects of thin film stiffness, material properties, loading depth, and surface/interface effect are depicted and discussed.

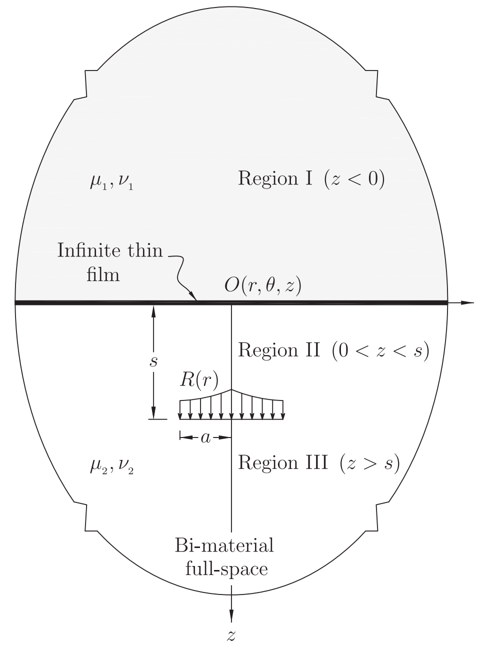

Fig. 1. A bi-material full-space reinforced by an extensible thin film at its interface.

2. Problem statement and governing differential equation

Consider a full-space solid comprising of two dissimilar linear elastic isotropic half-spaces perturbed by an extensible thin film of infinite extent and vanishingly small thickness at its interface, see Fig. 1. The origin of the cylindrical coordinate system $O(r, \theta, z)$ is set at the interface. The system is subjected to an axisymmetric normal load $R(r)$ acting on a finite region Π_s at the depth $z = s$.

For an elastic isotropic medium free of body forces and under the action of axisymmetric loading, the governing Cauchy–Navier equilibrium equation can be solved by employing the bi-harmonic potential function $\Phi(r, z)$ as (Love, 1927)

$$\nabla^2\nabla^2\Phi(r,z) = 0, \tag{1}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{2}$$

The components of the displacement vector, $\boldsymbol{u}$ and the Cauchy stress tensor, $\boldsymbol{\sigma}$ are given in terms of the potential function, Φ as (Love, 1927)

$$2\mu\, u_r = -\frac{\partial^2\Phi}{\partial r\partial z}, \tag{3}$$

$$2\mu\, u_z = \left[2(1-\nu)\nabla^2 - (3-2\nu)\frac{\partial^2}{\partial z^2}\right]\Phi, \tag{4}$$

$$\sigma_{rr} = \frac{\partial}{\partial z}\left[\nu\,\nabla^2 - (1+\nu)\frac{\partial^2}{\partial z^2}\right]\Phi, \tag{5}$$

$$\sigma_{\theta\theta} = \frac{\partial}{\partial z}\left[\nu\,\nabla^2 - \frac{1}{r}\frac{\partial}{\partial r} - \nu\frac{\partial^2}{\partial z^2}\right]\Phi, \tag{6}$$

$$\sigma_{zz} = \frac{\partial}{\partial z}\left[(2-\nu)\nabla^2 - (3-\nu)\frac{\partial^2}{\partial z^2}\right]\Phi, \tag{7}$$

$$\sigma_{rz} = \frac{\partial}{\partial r}\left[(1-\nu)\nabla^2 - (2-\nu)\frac{\partial^2}{\partial z^2}\right]\Phi, \tag{8}$$