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On weak and strong contact force networks in granular materials

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ABSTRACT

In an influential study on the micromechanical origin of stress transmission in granular materials, Radjaï et al. (1998) have proposed a division of the network of interparticle contacts into 'weak' and 'strong' contacts. This division is based on a comparison of the force at contacts with the average (over all contacts) force. They observed, from the results of a two-dimensional computer simulation of the behaviour of a system of particles, that the shear stress of the granular material is mainly carried by the strong contacts and that the anisotropy in the orientational distribution of the weak contacts is in the direction perpendicular to that of the full contact network.

These findings are analytically predicted here in a qualitative sense, within a statistical framework that is based on a simple, self-similar expression for the conditional probability function for the normal force at contacts with given contact orientation.

An alternative definition of weak and strong contacts is proposed here, in which the division of the contacts is based on a comparison of the force at contacts with the average force corresponding to the contact orientation. Contrary to the finding based on the definition of weak and strong contacts by Radjaï et al. (1998), with this alternative definition the pressure and the shear stress are (almost) equally carried by the weak contact network.

1. Introduction

Granular materials are systems consisting of a large number of particles with frictional interactions. Macroscopically, these systems exhibit a shear strength that is pressure dependent. Conventionally, this is described by the Mohr–Coulomb yield criterion at the macroscopic, continuum scale.

For granular materials consisting of stiff particles, the microscopic scale of interest is that of particles and interparticle contacts. Particles interact at these contacts through contact forces. In micromechanics of quasi-static deformation of granular materials, relationships are investigated between micro-scale characteristics of particles and contacts and macroscopic characteristics of stress and strain.

An important characteristic at the microscopic scale of contacts is the distribution of contact orientations. Computer simulations and results from experiments have demonstrated that this distribution is in general anisotropic (for example Biarez and Wiendieck, 1963; Oda, 1972a, 1972b, 1972c; Rothenburg and Bathurst, 1989), either due to the method of sample preparation (inherent anisotropy) or due to deformation (induced anisotropy).

The micromechanical origin of stress transmission in granular materials has been investigated from a number of viewpoints. Experiments using photo-elastic materials have demonstrated the importance of anisotropy at the microscopic scale in the distribution of contact orientations (for example Biarez and Wiendieck, 1963; de Josselin de Jong and Verruijt, 1969; Drescher and de Josselin de Jong, 1972; Oda, 1972a, 1972b, 1972c; Majmudar and Behringer, 2005). Rothenburg and Bathurst (1989) developed the stress–force–fabric relationship, which gives a quantitative relationship between macroscopic shear strength and microscopic anisotropies in contact orientations and in contact force distributions. Subsequent further developments of stress–force–fabric relationships have been reported by Ouadfel and Rothenburg (2001), Li and Yu (2013) and Azéma et al. (2013). A different, influential viewpoint on stress transmission in granular materials has been proposed by Radjaï et al. (1998). This viewpoint forms the focus of the current study.

Radjaï et al. (1998) have proposed to divide the set of interparticle contacts into two disjoint subsets, the 'weak' and the 'strong' contact networks. The weak contacts are defined as those contacts c at which the normal force f_n^c is smaller than a threshold value that depends on the average (over all contacts) normal force $\bar{F}$. For strong contacts c, the normal force f_n^c is larger than the average normal force $\bar{F}$. Thus

$$\text{Weak contacts}: \ f_n^c \leq \xi \bar{F} \quad \text{Strong contacts}: \ f_n^c > \xi \bar{F} \tag{1}$$

Here ξ is a dimensionless parameter through which a subset of contacts is identified.

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Based on results of a two-dimensional computer simulation of the behaviour of a system of particles, Radjaï et al. (1998) noted that:

1. The shear stress of the granular material is largely determined by the contributions of the strong force network.
2. The direction of anisotropy of the weak contact network is perpendicular to the direction of anisotropy of the full contact network.
3. Interparticle friction is mostly activated at weak contacts. Although Radjaï et al. (1998) relate the mobilisation of friction directly to dissipation, such a relation is more complex (see Kruyt and Rothenburg, 2006).

An objective of the current study is to provide an explanation of the first two findings, based on fairly weak assumptions that allow for analytical considerations.

Besides the strong and weak contact network of Radjaï et al. (1998), alternative contact networks have been proposed (see for example Kruyt and Antony, 2007; Tordesillas and Muthuswamy, 2009; Hunt et al., 2010).

The overview of this study is as follows. The relevant basics of micromechanics of granular materials are summarised in Section 2. The first two findings of Radjaï et al. (1998) are explained within a statistical framework for the contact forces in Section 3. An alternative definition of weak and strong contact networks is proposed in Section 4. Finally, findings of this study are discussed in Section 5.

2. Micromechanics

For two particles p and q that are in contact, the vector from the centre of particle p to the centre of particle q is the branch vector $\boldsymbol{l}^{pq}$. The unit vector corresponding to the branch vector is the contact normal $\boldsymbol{n}^{pq}$. In the two-dimensional case considered here, the orientation θ^c of a contact c is the angle of the contact normal vector $\boldsymbol{n}^c$ with respect to a reference direction.

Statistical properties of the contact orientations θ^c can be described by a fabric tensor (for example Satake, 1978; Kanatani, 1984) or by the contact distribution function $E(\theta)$ (Horne, 1965). The contact distribution function $E(\theta)$ gives the probability that for an arbitrary contact c its orientation θ^c lies in an interval of width $\Delta\theta$ around orientation θ

$$E(\theta)\Delta\theta = \mathrm{Prob}\left[\theta - \frac{\Delta\theta}{2} < \theta^c < \theta + \frac{\Delta\theta}{2}\right] \tag{2}$$

The force exerted by particle q on particle p is denoted by $\boldsymbol{f}^{pq}$. In the two-dimensional case, the force vector $\boldsymbol{f}^c$ at a contact c can be decomposed into scalar normal and tangential components, f_n^c and f_t^c respectively.

The expression for the average Cauchy stress tensor $\boldsymbol{\sigma}$, in terms of contact force vectors $\boldsymbol{f}^c$ and branch vectors $\boldsymbol{l}^c$, is given by (for example Drescher and de Josselin de Jong, 1972; Kruyt and Rothenburg, 1996)

$$\sigma_{ij} = \frac{1}{A}\sum_{c\in C} f_i^c l_j^c \tag{3}$$

where the summation is over contacts c in the set of contacts C that are present in the region of interest with area A (in the two-dimensional case considered here).

By grouping contacts with similar contact orientations, the discrete sum in Eq. (3) can be converted to an integral involving the contact distribution function $E(\theta)$ (Rothenburg and Bathurst, 1989)

$$\sigma_{ij} = m_A \int_0^{2\pi} E(\theta)\overline{f_i l_j}(\theta)d\theta \tag{4}$$

where $m_A = N_{\mathrm{cont}}/A$ is the contact density (i.e. the number of contacts per unit area) and $\bar{\phi}(\theta)$ denotes the average of an arbitrary contact quantity ϕ^c over contacts with similar contact orientations θ.

Eq. (4) can be simplified by adopting two assumptions that have been verified by Rothenburg and Bathurst (1989). Firstly, it is assumed that the contact force vector $\boldsymbol{f}^c$ and the branch vector $\boldsymbol{l}^c$ are uncorrelated, i.e. $\overline{f_i l_j}(\theta) \cong \overline{f_i}(\theta)\overline{l_j}(\theta)$. Secondly, for disk-shaped particles as considered here, the average branch vector is given by $\overline{l_j}(\theta) \cong \bar{D} n_j(\theta)$ where $\bar{D}$ is the average particle diameter. With these assumptions, Eq. (4) can be simplified to

$$\sigma_{ij} = m_A \bar{D} \int_0^{2\pi} E(\theta)\overline{f_i}(\theta)n_j(\theta)d\theta \tag{5}$$

3. Analysis of findings by Radjaï et al. (1998)

The analysis by Radjaï et al. (1998) of the stress tensor $\boldsymbol{\sigma}$ is based on a division of the contact into weak and strong contacts, depending on the magnitude of the normal force f_n^c. Here this concept is formally described by a probability density function for the contact forces. To emphasise the main ideas and to allow for simple analytical developments, the contribution of the tangential forces f_t^c to the stress tensor is neglected here. Rothenburg and Bathurst (1989) have shown that the contribution of tangential forces to the macroscopic shear strength is not dominant.

The *joint* probability density function $P(f_n, \theta)$ describes the probability that for an arbitrary contact c its contact normal force f_n^c lies within an interval of width Δf_n around f_n *and* its contact orientation θ^c lies within an interval of width $\Delta\theta$ around θ. Thus

$$\mathrm{Prob}\left[f_n - \frac{\Delta f_n}{2} < f_n^c < f_n + \frac{\Delta f_n}{2}, \theta - \frac{\Delta\theta}{2} < \theta^c < \theta + \frac{\Delta\theta}{2}\right] = P(f_n, \theta)\Delta f_n \Delta\theta \tag{6}$$

Alternatively, a *conditional* probability density function $P(f_n|\theta)$ can be defined that gives the conditional probability that for a contact c whose contact orientation θ^c lies within an interval of width $\Delta\theta$ around θ, its contact normal force f_n^c lies within an interval of width Δf_n around f_n. Thus

$$\mathrm{Prob}\left[f_n - \frac{\Delta f_n}{2} < f_n^c < f_n + \frac{\Delta f_n}{2} \middle| \theta - \frac{\Delta\theta}{2} < \theta^c < \theta + \frac{\Delta\theta}{2}\right] = P(f_n|\theta)\Delta f_n \Delta\theta \tag{7}$$

The joint and conditional probability density functions, $P(f_n, \theta)$ and $P(f_n|\theta)$ respectively, are related by

$$P(f_n, \theta) = E(\theta)P(f_n|\theta) \tag{8}$$

Since $P(f_n|\theta)$ is a (conditional) probability density function, it must satisfy the normalisation condition for probabilities

$$1 = \int_0^\infty P(f_n|\theta)df_n \tag{9}$$

In terms of the conditional probability density function $P(f_n|\theta)$, the average normal force $\overline{f_n}(\theta)$ for contacts with orientation θ is given by

$$\overline{f_n}(\theta) = \int_0^\infty f_n P(f_n|\theta)df_n \tag{10}$$

The average, over all contacts, normal force $\bar{F}$ is then given by

$$\bar{F} = \int_0^{2\pi} E(\theta)\overline{f_n}(\theta)d\theta \tag{11}$$

In biaxial and isobaric tests the contact distribution function $E(\theta)$ and the average normal force $\overline{f_n}(\theta)$ are well described by a