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Adhesive contact between a rigid spherical indenter and an elastic multi-layer coated substrate

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ABSTRACT

In this paper the frictionless, adhesive contact between a rigid spherical indenter and an elastic multi-layer coated half-space was investigated by means of an integral transform formulation. The indented multi-layer coats were considered as made of isotropic layers that are perfectly bonded to each other and to an isotropic substrate. The adhesive interaction between indenter and contacting surface was treated as Maugis-type adhesion to provide general applicability within the entire range of adhesive interactions. By using a transfer matrix method, the stress–strain equations of the system were reduced to two coupled integral equations for the stress distribution under the indenter and the ratio between the adhesion radius and the contact radius, respectively. These resulting integral equations were solved through a numerical collocation technique, with solutions for the load dependencies of the contact radius and indentation depth for various values of the adhesion parameter and layer composition. The method developed here can be used to calculate the force-distance response of adhesive contacts on various inhomogeneous half-spaces that can be modeled as multi-layer coated half-spaces.

1. Introduction

The importance of contact mechanics at small scales gained new relevance from the perspective of its ubiquitous presence and usage in micrometer- and nanometer-scale applications and measurements. As an example, micro- and nano-electromechanical systems include parts that use contact mechanics to perform their operations. As such, their life-time functionality relies on designs that make use of contact mechanics and stay free of its pervasive effects (e.g. adhesion, fatigue, etc.). It is also important to have precise and accurate characterization of the mechanical properties of the materials and structures of these devices. The methods and techniques developed for micrometer and nanometer-scale mechanical property characterization (e.g. instrumented indentation testing, atomic force microscopy) operate also on contact mechanics and therefore proper understanding of the mechanics involved in measurements is necessary to extract the mechanical properties of the materials tested.

Important elastic, viscous, and plastic properties of materials at micrometer and nanometer scales are retrieved from indentation-type measurements that directly probe the contact mechanical response of materials. In general, these measurements (e.g. instrumented indentation testing Oliver and Pharr, 1992 and force-distance by atomic force microscopy Radmacher et al., 1994; Butt et al., 2005) provide control and measurement of the contact force between an indenter and the surface probed and the relative deformation sustained by indenter and surface. These two measured quantities, applied force and indentation depth, are then used in a contact mechanical model to inversely calculate parameters like elastic modulus, hardness, adhesive forces, etc. It is thus important to understand and be able to solve the contact mechanics involved in such measurements.

The first contact mechanics model proposed by Hertz in 1882 captures the mechanics of the contact between two spherical elastic bodies (e.g. Johnson, 1985). Since then, the Hertz model was successfully used to describe frictionless, non-adhesive contacts between an indenter (rigid or elastic) and the surface of a half-space material. The problem consists of solving the stress and deformation fields in elastic half-space solids subjected to a particular load distributed over the contact area (e.g. Love, 1906; Gladwell 1980; Johnson, 1985). However, for many cases of interest (microelectronics, coatings, electromechanical devices, composites, etc.), this basic contact problem requires extension to the study of indentation on coated substrates, in which case the elastic field

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within the coated layered half-space and stress distribution on the surface need to be solved for. Fully analytical solutions for the frictionless, non-adhesive contact problem on single- and multi-layer coated substrates were achieved by the integral transform method (Gupta and Wallowit, 1974; Yu et al., 1990; Stone, 1998; Korsunsky and Constantinescu, 2009; Constantinescu et al., 2013) and the method of images (Schwarzer, 2000; Fabrikant, 2006). Numerical solutions to the Hertz contact problem on single- and multi-layer coated substrates were obtained in various studies by means of finite element methods (Djabella and Arnell, 1992; Bouzakisa et al., 2001; Zhao et al., 2011).

It was recognized also that, in many cases, the adhesive forces have to be incorporated in the contact model used to account for the surface energies of the two bodies in contact (Bradley, 1932; Johnson et al., 1971; Derjaguin et al., 1975). By considering a uniform tensile stress within an adhesive annular region surrounding the contact zone, Maugis (1992) proposed an adhesive contact model with general applicability through the entire range of the adhesive interactions. The Maugis model provides a continuous transition between the two established adhesive contact limits: the JKR approximation (Johnson et al., 1971) for contacts on compliant materials with large surface energy and large contact radius and the DMT approximation (Derjaguin et al., 1975) for contacts on stiff materials with low surface energy and small contact radius, respectively. Notably, other proposed adhesive contact models capture the JKR-DMT transition (Greenwood and Johnson, 1998; Schwarz, 2003).

Owning to various measurement configurations (surface force apparatus, atomic force microscopy) and sample geometries (thin films on substrates), extensions of the adhesive contact models to adhesive contacts on coated substrates were proposed. Thus, finite element (Johnson and Sridhar, 2001; Sridhar and Sivashanker, 2003; Sridhar et al., 2004) and semi-analytical solutions based on integral transform approaches were developed for adhesive contacts on coated elastic substrates (Mary et al., 2006; Sergici et al., 2006; Barthel and Perriot, 2007; Choi, 2012). Although most of the above models were developed as extensions of the JKR theory, some models explicitly captured the JKR-DMT transition for adhesive contacts on coated substrates (Sergici et al., 2006). Currently there is no analytical adhesive contact model for adhesive contacts on multi-layer coated substrates.

In this paper the frictionless, adhesive contact between a rigid spherical indenter and an elastic multi-layer coated half-space is investigated by means of an integral transform formulation initially developed by Civelek and Erdogan (1974) for the axisymmetric double contact problem of an elastic layer pressed against of a half-space by an elastic stamp. The method was adapted by Sergici et al. (2006) to the contact between a spherical indenter and a single-layer coated substrate and is extended here to the case of indented multi-layer coated substrates. In this context, the adhesive interaction between the indenter and the coated surface is considered as Maugis-type adhesion to provide general applicability within the entire range of adhesive interactions. The indented multi-layer coat is made of isotropic layers that are perfectly bonded to each other and to an isotropic substrate. The elastic displacements and stresses in the coating layers and substrate are solved in terms of Papkovich–Neuber potentials and, through a transfer matrix method, used to express the surface displacement inside the contact zone and the stress at the free surface outside the contact zone. The obtained mixed boundary equations are reduced to two coupled integral equations for the stress distribution induced by the indenter on the multi-layer coated substrate and the ratio between the adhesion radius and the contact radius, respectively. These resulting integral equations are solved through a numerical collocation technique, with solutions for the load dependencies of the contact radius and indentation depth for various values of the adhesion parameter and layer composition.

The model developed here for an adhesive contact on a multi-layer coated substrate can be readily applied to adhesive contacts on inhomogeneous half-spaces that can be approximated as a series of isotropic layers (e.g. compositionally graded materials).

2. Contact mechanics model

The contact geometry considered in this study is shown in Fig. 1, in which a spherical rigid indenter of radius R is brought into contact with a multi-layer coated substrate. The coat of thickness h is made of n different layers which are perfectly bonded to each other and to the substrate; all the layers and the substrate are assumed isotropic and perfectly elastic. The contact was considered frictionless and the adhesion between the contacting surfaces was modeled as a Maugis-type adhesion (Maugis, 1992). This adhesive interaction assumes a constant tensile stress σ_0 acting outside the contact zone where the contacting surfaces are separated by a distance less than h_0. The adhesive stress σ_0 in the annular cohesive region ($a < r < c$; a is the contact radius and c is the adhesion radius) complements the surface traction that occurs inside the actual contact zone ($r < a$). As such, the surface deforms under the action of the external load (the applied force) and the internal load (the stress σ_0 in the gap between indenter and surface). The surface displacement equates the deformation of the layer coated substrate, in accordance with the mechanical properties of the layers and substrate and the stress transfer at their interfaces. In the following the mixed stress–strain relations in the layers and substrate are examined to determine the stress distribution under the indenter.

The elastic displacements and stress fields in each isotropic material (layers and substrate) are expressed in terms of the harmonic axisymmetric Papkovich–Neuber potentials, $\Psi = (0, 0, \Psi_z)$ and Φ:

$$2\mu_{(i)} u_r^{(i)} = z\frac{\partial \Psi_z^{(i)}}{\partial r} + \frac{\partial \Phi^{(i)}}{\partial r},$$

$$2\mu_{(i)} u_z^{(i)} = z\frac{\partial \Psi_z^{(i)}}{\partial z} - (3-4\nu_{(i)})\Psi_z^{(i)} + \frac{\partial \Phi^{(i)}}{\partial z},$$

$$\sigma_{zz}^{(i)} = z\frac{\partial^2 \Psi_z^{(i)}}{\partial z^2} - 2(1-\nu_{(i)})\frac{\partial \Psi_z^{(i)}}{\partial z} + \frac{\partial^2 \Phi^{(i)}}{\partial z^2},$$

$$\sigma_{zr}^{(i)} = \frac{\partial}{\partial r}\left[z\frac{\partial \Psi_z^{(i)}}{\partial z} - (1-2\nu_{(i)})\Psi_z^{(i)} + \frac{\partial \Phi^{(i)}}{\partial z}\right], \quad (1)$$

where the index i counts the layers from $i = 1$ for the top layer (free surface at $z = -h$) to $i = n$ for the bottom layer (on top of the substrate) and also the substrate $i = n + 1$ (as shown in Fig. 1, the top of the substrate is at $z = 0$). The elastic constants used in (1) are in terms of shear moduli $\mu_{(i)}$ and Poisson ratios $\nu_{(i)}$ of the materials. Alternatively, we can use Young's moduli $E_{(i)} = 2\mu_{(i)}(1+\nu_{(i)})$ and Poisson ratios $\nu_{(i)}$ to relate them with the plane strain moduli (indentation moduli) $E_{(i)}/(1-\nu_{(i)}^2)$ of the materials.

The partial differential equations (1) can be converted into ordinary differential equations by means of the Hankel transforms of the Papkovich–Neuber potentials. The Hankel transforms of order zero, $(\overline{\Psi^{(i)}}, \overline{\Phi^{(i)}})$, of the Papkovich–Neuber potentials for each layer i (i from 1 to n) and the substrate ($i = n+1$) are found as solutions of their Hankel transformed Laplacian equations:

$$\overline{\nabla^2\Psi^{(i)}} = \left(\frac{d^2}{dz^2} - p^2\right)\overline{\Psi^{(i)}} = 0,$$

$$\overline{\nabla^2\Phi^{(i)}} = \left(\frac{d^2}{dz^2} - p^2\right)\overline{\Phi^{(i)}} = 0. \quad (2)$$