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Nonlinear vibration of dielectric elastomer incorporating strain stiffening

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ABSTRACT

Due to the strain-stiffening of polymer chains, a membrane of dielectric elastomer (DE) can reach two different stable equilibrium states under a static electrical load. In this paper, a theoretical model is developed to investigate the strain-stiffening effect on the nonlinear vibration of a circular DE membrane subjected to electro-mechanical loading. Free vibration, steady parametric excitation and chaos of the DE membrane undergoing large deformation are studied respectively. We find that after a small perturbation the DE membrane vibrates steadily around the two stable stretches and two natural frequencies exist for the same loading condition. With the increase of initial perturbation energy, the amplitude–frequency response of free vibration shows a transition from behaving like a soft spring to a hard spring attributed to strain-stiffening effect. When driven by a sinusoidal voltage, the DE membrane can resonate at multiple frequencies of excitation around small and large stable equilibrium states respectively. Variation of the sinusoidal voltage may induce a sudden change from steady vibration to chaos and the critical conditions for the transition are numerically calculated.

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1. Introduction

As a promising material for the artificial-muscle technology, dielectric elastomer (DE) has received considerable attention in recent years. The DEs have excellent attributes including large voltage-induced strains, light weight, high energy density, fast response, silent operation, low maintenance, and low cost (Bauer et al., 2014; Brochu and Pei, 2010; Carpi et al., 2010; Pelrine et al., 2000; Wang et al., 2014). Subject to a voltage through the thickness, the DE membrane coated on both surfaces with compliant electrodes reduces in thickness and expands in area. The typical applications of DE transducers are soft robots (Anderson et al., 2012; Bar-Cohen, 2004; Nguyen et al., 2014), adaptive optics (Carpi et al., 2011; Hanley et al., 2014; Shian et al., 2013; Wei et al., 2014), generators (Huang et al., 2013; Kornbluh et al., 2012; McKay et al., 2015), sensors (Girard et al., 2015; Noda et al., 2014), resonators (Dubois et al., 2008), loudspeakers (Heydt et al., 2000; Keplinger et al., 2013) and active vibration isolation (Jones and Sarban, 2013).

For DE transducers utilizing the dynamic properties of DEs, such as resonators, the effect of inertia must be taken into account. A number of studies on nonlinear dynamic performances of DEs have been conducted, mostly focusing on the behaviors around small stable equilibrium stretch with small amplitude (Fox and Goulbourne, 2008; Li et al., 2012; Park et al., 2012; Sheng et al., 2014; Son and Goulbourne, 2010; Zhu et al., 2010a, b). No works address the nonlinear vibration around large stable equilibrium stretch considering the strain-stiffening effect.

When we stretch a rubber, it is easy to stretch it at first, but if the stretch is large enough, it will be increasingly hard to stretch it further. This can be explained from the microscopic picture: The rubber consists of polymer chains, each of which has a finite contour length. When we stretch the rubber, the polymer chains are elongated. With the increase of the stretch, the end-to-end distance of each polymer chain may approach the finite contour length. This will result in the steep strain-stiffening. Hence when a DE membrane undergoes large deformation, the strain-stiffening effect should be considered.

Taking account of the strain-stiffening effect, many researchers have studied the behaviors of DE membranes under quasi-static loading both theoretically and experimentally (An et al., 2015; Huang and Suo, 2011; Keplinger et al., 2012; Li et al., 2013; Lu and Suo, 2012; Zhao et al., 2007; Zhou et al., 2008). It is reported that strain-stiffening enables the DE membrane to achieve giant voltage-induced deformation after the snap-through instability (Keplinger et al., 2012; Li et al., 2013). Furthermore, the phenomenon of electromechanical phase transition has been observed in a recent experiment due to the strain-stiffening of DE

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membrane, where a huge voltage-induced areal expansion up to 2200% was achieved (An et al., 2015). However, the significant strain-stiffening effect on the dynamic characteristics of a DE membrane is unknown.

In this paper, a theoretical model is developed to investigate the strain-stiffening effect on the nonlinear vibration of a circular DE membrane subjected to an equal biaxial dead force and a time-dependent voltage through the thickness. The paper is organized as follows. Section 2 derives the governing equations. Section 3 introduces the small perturbation around the static equilibrium states. Section 4 studies the large perturbation induced free vibrations around the stable equilibrium states. Section 5 shows the steady parametric excitation with sinusoidal voltages, especially for the cases around large stable equilibrium states. Section 6 analyzes the electro-mechanical stability of the double-well vibration, and calculates the critical conditions for the transition from steady vibration to chaos.

2. Governing equations

In this section, the governing equations for a circular DE membrane subject to equal biaxial dead force and a time-dependent voltage are derived by using the Euler–Lagrange equation. The strain-stiffening effect is taken into account by employing the Gent material model (Gent, 1996).

Fig. 1 shows a circular DE membrane sandwiched between two compliant electrodes. In the reference state, the DE membrane of thickness H and radius R_0 is un-deformed, and each material particle in the membrane is labeled by its distance R from the center O, as shown in Fig. 1(a). In the actuated state, the membrane is subject to a constant radial force P uniformly distributed along the hoop direction, and the two electrodes are subject to a time-dependent voltage $\Phi(t)$ through a conducting wire, as shown in Fig. 1(b). Electrons flow through the conducting wire from one electrode to the other, and the two electrodes gain charges $+Q(t)$ and $-Q(t)$, respectively. Due to the attraction of charges of opposite signs $Q(t)$ as well as the radial force P, the DE membrane deforms uniformly to a configuration of thickness h and radius r_0 at time t. As a plane axisymmetric problem, the circular DE membrane undergoes homogeneous equal-biaxial deformation, with the stretch $\lambda(t) = r_0(t)/R_0$. Hence, the material particle R occupies a place of coordinate $r(R,t) = \lambda(t)R$ in the actuated state. The function $r(R, t)$ specifies the time-dependent deformation of the membrane.

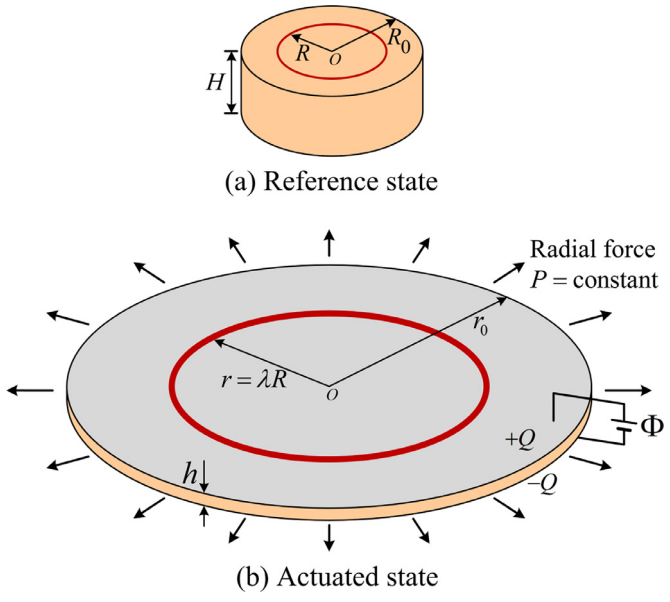

Fig. 1. A circular membrane of dielectric elastomer deforms at two states. (a) The reference state, the membrane is subject to no force and no voltage, (b) the actuated state, the membrane is subject to a dead radial force uniformly distributed along the hoop direction and a time-dependent voltage through the thickness.

The elastomer is taken to be incompressible, so that $(r_0(t))^2 h(t) = R_0^2 H$, we have $h(t) = (\lambda(t))^{-2} H$. The time-dependent equal-biaxial stresses can be written as $\sigma_r(t) = \sigma_\theta(t) = P/(2\pi r_0(t) h(t)) = \lambda(t) P/(2\pi R_0 H)$, where $\sigma_r(t)$ and $\sigma_\theta(t)$ are the radial stress and the hoop stress, respectively. Consider the DE membrane coated with electrodes as a deformable capacitor. The true electric field is $E(t) = \Phi(t)/h(t) = (\lambda(t))^2 \Phi(t)/H$, and the true electric displacement is $D(t) = Q(t)/(\pi (r_0(t))^2)$. Thus the magnitude of charge on either electrode can be written as $Q(t) = \pi (\lambda(t))^2 R_0^2 D(t)$.

The circular DE membrane, along with the mechanisms that apply the time-dependent voltage and the dead radial force, constitutes a composite thermodynamic system. We suppose the composite exchanges energy with the rest of the world by heat, but is held under an isothermal condition. Moreover, if we neglect the viscosity of the material, the composite has two independent variables $\lambda(t)$ and $D(t)$. For an equilibrium state, the kinetic energy E_k and the Helmholtz free energy Π of the composite system should satisfy the Euler–Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v_j}\right) - \frac{\partial L}{\partial x_j} = 0, \quad j = 1, 2, \tag{1}$$

where $L = E_k - \Pi$ is the Lagrangian, x_j denotes the two independent variables, $x_1 = \lambda$ and $x_2 = D$, v_j is the derivative of x_j with respect to time t.

Consider an annular material element between two circles of material particles with radius R and $R + dR$. When the DE membrane is in the actuated state at time t, the circle of material particles of radius R becomes a circle of radius $r(R, t)$, and the other circle of material particles, radius $R + dR$, becomes a circle of radius $r(R + dR, t)$. The annular material element of width dR in the reference state deforms to an annulus of width $dr = \lambda(t) dR$. During actuation, the mass of the annular material element, marked by the position coordinate $r(R, t)$, is $dm = 2\pi \rho h r dr$, where ρ denotes the density of the DE membrane, and the velocity of the element can be written as $\partial r(R,t)/\partial t = R d\lambda(t)/dt = R v_1$. Thus, the kinetic energy of the composite can be integrated along the radial direction of the DE membrane:

$$E_k = \int_0^{r_0} \frac{1}{2}(R v_1)^2 dm = \int_0^{R_0} \pi \rho H R^3 v_1^2 dR = \frac{\pi \rho H R_0^4 v_1^2}{4}. \tag{2}$$

The Helmholtz free energy of the composite is the sum of the free energy of the membrane and the potential energy of the mechanisms that apply voltage and force:

$$\Pi = \pi R_0^2 H W(\lambda, D) - \Phi Q - P r_0, \tag{3}$$

where $W(\lambda, D)$ is the Helmholtz free energy density of the DE membrane defined by the total free energy in the deformed state divided by the volume in the reference state.

Substituting Eqs. (2) and (3) into Eq. (1), we obtain the governing equations

$$\frac{\rho R_0^2}{2}\frac{d^2\lambda}{dt^2} + \frac{\partial W(\lambda, D)}{\partial \lambda} - \frac{2\lambda \Phi D}{H} - 2s = 0, \tag{4}$$

and

$$\frac{\partial W(\lambda, D)}{\partial D} = E, \tag{5}$$

where $s = P/(2\pi R_0 H)$ is the nominal equal-biaxial stress.

The governing Eqs. (4) and (5) are valid for an arbitrary material model specified by the free energy function $W(\lambda, D)$. In

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