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A homogenization method for geometric nonlinear analysis of sandwich structures with initial imperfections

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ABSTRACT

A homogenization method for geometric nonlinear analysis of structural core sandwich panels is proposed. The method provides high computational performance based on an efficient separation of scales. In the macroscale, the sandwich panel is discretized with an equivalent single layer of shell elements. The macroscale shell stiffness matrix is nonlinear, obtained from the analysis of a representative volume element. Prescribed displacement boundary conditions are applied to the representative unit based on the strain definitions of the first-order shear deformation theory. Changes in local wavelength in the post-buckling are considered in the analyses. Manufacturing-induced imperfections are introduced to local and global scales. The method allows for description of buckling in these two scales and is shown to hold good accuracy with respect to equivalent 3D FEM models. Examples include web-core and corrugated-core sandwich panels. The method can be extended to any periodic structure of complex local topology. It can be easily implemented to commercial FE packages.

1. Introduction

Sandwich panels are widely used in structural design, mainly in applications where the stiffness-to-weight ratio is critical. They are typically composed of thin face sheets connected to a low density core by adhesion, welding or riveting. The core topology and material depend on the application purpose. In structural core sandwich panels, the core is typically visibly discrete enabling an extremely lightweight assembly with possibility to integrate non-structural functions (Valdevit et al., 2004). Due to their discrete local geometry, modelling large sandwich structures with the 3D FEM is often cumbersome. The difficulties yet increase when nonlinearities are introduced. High computational and modelling efforts required can turn the approach unfeasible if design cycles are needed to be fast. Thus, analysis of structures composed of sandwich panels has long been performed in terms of effective properties (Noor et al., 1996).

Linear elastic effective mechanical properties of sandwich panels have been derived analytically and computationally by a number of authors (Fung et al., 1994; Libove and Batdorf, 1948; Libove and Hubka, 1951; Romanoff and Varsta, 2006; Romanoff and Varsta, 2007; Xia et al., 2012; Xu and Qiao, 2002); see Hohe and Becker (Hohe and Becker, 2002) for a comprehensive review. In several engineering applications however, geometric nonlinearity plays an important role in the structural response. As structural core sandwich assemblies are often composed of thin plates, buckling can happen well before material failure.

Investigations on the geometric nonlinear behaviour of homogenized sandwich panels often rely on analytical approximations. While several studies predict buckling loads, few authors attempted to describe their post-buckling behaviour. Nordstrand, (2004) developed a theoretical expression for global buckling and post-buckling analysis of corrugated boards. Jelovica and Romanoff, (2013) studied web-core sandwich panels under compression using the equivalent single layer (ESL) theory with analytical stiffnesses. These studies did not take local manufacturing imperfections (SANDWICH consortium, 2003) or local buckling at the unit cell level into consideration. Local buckling is however known to be critical in the nonlinear response of sandwich structures (Jelovica and Romanoff, 2015). Byklum and Amdahl, (2002) and Byklum et al., (2004) established a semi-analytical method for buckling and post-buckling assessment of single-sided stiffened panels. A local model is used to determine the nonlinear part of the stiffness based on pre-defined deformation modes. Thereafter, the local buckling is smeared over the entire panel domain. Rabczuk et al., (2004) proposed a numerical method in which the sandwich assembly is reduced to a discrete system of interlaced beams and shells. The method is shown for the 2D case and extension to 3D requires considerable effort.

Recently, multiscale homogenization became an alternative to model heterogeneous assemblies (Geers et al., 2010), such as sandwich panels. Helfen and Diebels, (2011) proposed a FE^2 method for sandwich plates with simplified tangent stiffness derivation.

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Geers et al., (2007) and Coenen et al., (2010) developed concurrent homogenization schemes for thin sheets based on a second order technique. Given the high computational cost of these methods and the fact that real structures have manufacturing-induced initial imperfections, there is room for a method that uses predefined shape as initial state of the simulations.

This work presents a multiscale homogenization scheme for the analysis of sandwich structures. It extends the work of Jelovica and Romanoff, (2013) using a similar concept by Byklum et al., (2004), in which the shell stiffnesses are progressively modified to describe local nonlinearity. The scope is restricted to panels with discrete core, assembled with layers of isotropic material and relatively thin faces. The method can describe geometric nonlinear effects, in particular global and local post-buckling, with excellent computational efficiency thanks to an efficient separation of scales. It is assumed that elastic strain energy has to increase for increasing load, both at global and local scales (see Miehe et al., (2002)). The shell stiffnesses are determined from a representative volume element (RVE) and described through nonlinear *stress resultant* vs. *strain* relations. The stiffnesses are combined to the shell element stiffness matrix according to the macroscale strain-state. The macroscale behaviour is based on the first order shear deformation theory.

Section 2 introduces the macroscale shell kinematics and constitutive equations. The transition between scales is presented. Section 3 deals with the microscale response and choice of local RVE boundary conditions. Section 4 presents two numerical examples; corrugated and web-core sandwich structures are chosen based on their practical relevance and susceptibility to buckling.

2. Macroscale response and micro–macro transition

2.1. Shell kinematics and constitutive equations

The macroscale shell behaviour follows a first-order shear deformation theory for laminates, e.g. (Reddy, 2000); see Fig. 2.1. The displacement field associated with the shell continuum is defined as

$$\begin{aligned} u &= u^0 + x_3\phi_1 \\ v &= v^0 + x_3\phi_2 \\ w &= w^0 \end{aligned} \tag{1}$$

where the superscript ^{0} refers to the shell mid-plane. The rotations of the transverse normals are given by

$$\phi_1 = \frac{\partial u}{\partial x_3}, \quad \phi_2 = \frac{\partial v}{\partial x_3}, \tag{2}$$

The nonlinear strain field with *von Karman* assumptions is divided into extensional $^{(0)}$ and bending $^{(1)}$ components as

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \varepsilon_{11}^{(0)} \\ \varepsilon_{22}^{(0)} \\ \gamma_{23}^{(0)} \\ \gamma_{13}^{(0)} \\ \gamma_{12}^{(0)} \end{bmatrix} + x_3 \begin{bmatrix} \varepsilon_{11}^{(1)} \\ \varepsilon_{22}^{(1)} \\ \gamma_{23}^{(1)} \\ \gamma_{13}^{(1)} \\ \gamma_{12}^{(1)} \end{bmatrix} = \begin{bmatrix} \frac{\partial u^0}{\partial x_1} + \frac{1}{2}\left(\frac{\partial w^0}{\partial x_1}\right)^2 \\ \frac{\partial v^0}{\partial x_2} + \frac{1}{2}\left(\frac{\partial w^0}{\partial x_2}\right)^2 \\ \frac{\partial w^0}{\partial x_2} + \phi_2 \\ \frac{\partial w^0}{\partial x_1} + \phi_1 \\ \frac{\partial u^0}{\partial x_2} + \frac{\partial v^0}{\partial x_1} + \frac{\partial w^0}{\partial x_1}\frac{\partial w^0}{\partial x_2} \end{bmatrix} + x_3 \begin{bmatrix} \frac{\partial \phi_1}{\partial x_1} \\ \frac{\partial \phi_2}{\partial x_2} \\ 0 \\ 0 \\ \frac{\partial \phi_1}{\partial x_2} + \frac{\partial \phi_2}{\partial x_1} \end{bmatrix} \tag{3}$$

The shell constitutive behaviour is defined in terms of extensional (A), bending (D) and coupling:extensional-bending (B) and bending-extensional (C) stiffness matrices. The relation between stress resultant and strains is given by

$$\begin{bmatrix} N_{11} \\ N_{22} \\ N_{12} \\ M_{11} \\ M_{22} \\ M_{12} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & C_{11} & C_{12} & 0 \\ A_{21} & A_{22} & 0 & C_{21} & C_{22} & 0 \\ 0 & 0 & A_{33} & 0 & 0 & C_{33} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{21} & B_{22} & 0 & D_{21} & D_{22} & 0 \\ 0 & 0 & B_{33} & 0 & 0 & D_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{11}^{(0)} \\ \varepsilon_{22}^{(0)} \\ \gamma_{12}^{(0)} \\ \varepsilon_{11}^{(1)} \\ \varepsilon_{22}^{(1)} \\ \gamma_{12}^{(1)} \end{bmatrix} \tag{4}$$

And the shear terms

$$\begin{bmatrix} Q_2 \\ Q_1 \end{bmatrix} = \begin{bmatrix} KA_{44} & 0 \\ 0 & KA_{55} \end{bmatrix} \begin{bmatrix} \gamma_{23} \\ \gamma_{13} \end{bmatrix} \tag{5}$$

where K is shear-correction factor and A_{44} and A_{55} the out-of-plane shear stiffnesses.

2.2. Micro–macro transition

The stiffness matrix coefficients (of Eq. 4) are obtained from nonlinear microscale analyses. A RVE of the periodic structure is defined and subjected to prescribed displacement boundary conditions (see for details Section 3). The RVE includes local initial imperfections. The boundary conditions are chosen so the RVE is subjected to a single shell equivalent strain component at a time. Six models, one for each component of the strain vector (Eq. 4), are necessary to determine 20 ABCD stiffnesses. *Stress resultant* vs. *strain* curves are computed based on average stresses and strains.

These curves are then used for the macroscale analysis. A stiffness component at a given global load step is determined by considering the tangent of the curve at the points of current macroscale shell strain ε_n and strain increment ε_{n+1}. Fig. 2.2 shows a summary of the method with two examples of boundary conditions.

Based on Fig. 2.2, the following steps can be identified:

1. RVE definition (shape, size, etc.).
2. Imposition of prescribed displacement boundary conditions based on shell strain components.
3. Nonlinear RVE analyses. Equivalent shell resultants for *m* nonlinear steps. Determination of *stress resultant* vs. *strain* relations.
4. Macroscale analysis (ESL model). Interpolation of stiffnesses for *n* nonlinear steps and assemblage of shell stiffness matrix.

An implementation example on a commercial FE package is summarised in Appendix A.

2.3. Initial imperfections

The initial imperfections are sinusoidal in shape (Fig. 2.3); see for measured values (SANDWICH consortium, 2003). Global panel imperfections are typically described with one half-wave over panel length and width, while the local imperfections have multiple waves in the panel domain (faces and core). The nodal points of the local imperfections are at the face-core intersections. The two imperfection types are linearly superimposed, see Fig. 2.2.