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Compression of unbonded rubber layers taking into account bulk compressibility and contact slip at the supports

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ABSTRACT

The behavior of rubber layers under pure compression has been investigated to considerable extent in the literature. The most widely used approach is the so-called *pressure solution*, which is based on several assumptions, most notably that the stress state is dominated by the hydrostatic pressure. Other approaches have also been considered, but for nearly incompressible material and thin layers their predictions are very similar to those of the pressure solution. Nearly all past studies on the subject have focused on rubber layers that are bonded to either rigid or flexible supports (or reinforcement). Unreinforced (i.e., single layer) rubber pads are often installed as unbonded, i.e., without steel end plates connecting them to their top and bottom supports. In an unbonded application, rubber pads rely solely on friction to develop shear resistance along the contact interfaces. This shear resistance is necessary to provide the pad with an adequately large vertical stiffness. The effect of the frictional restraint along the top and bottom contact surfaces and the influence of partial slip have received very little attention in the literature. In this paper, we present a theoretical analysis for the behavior of an unbonded rubber layer, including the effects of the elastomer's bulk compressibility and the contact slip at the supports. Results of a finite element analysis are also presented and shown to be in good agreement with the results of the theoretical analysis.

1. Introduction

Rubber bearings are used in a broad range of engineering applications, including buildings, bridges, storage tanks, railways, etc. Early applications date back to mid-nineteenth century when 50-mm thick rubber mats were installed to reduce railway vibration on the Britannia and Conwy Bridges in Wales (Ab-Malek and Roberts, 2013). Over time, the use of rubber bearings grew and extended to various new applications; most notably, they are currently used widely to accommodate deformations associated with thermal expansion/contraction, traffic loads, and construction misalignment in bridges (Stanton and Roeder, 1982; Constantinou et al., 2011), to isolate equipment and structures from vibration and shock (Snowdon, 1979), and to seismically isolate structures (Naeim and Kelly, 1999; Constantinou et al., 2007; Kelly and Konstantinidis, 2011).

The first attempt to predict the compression stiffness of a rubber layer bonded to rigid supports was made by Rocard (1937) using an energy approach. Further developments were made by Gent and Lindley (1959) who derived expressions for the compressive stiffness of long-strip and circular elastic layers bonded to rigid plates, assuming incompressible material. Gent and Meinecke (1970) extended the analysis and presented an expression for the compression modulus of a square-shape elastic layer. Lindley (1979) applied the energy method to extend the theory for incompressible material to compressible elastic layers.

The approach widely used to estimate the compression stiffness of rubber layers bonded to rigid supports originates from the work of Gent and Lindley (1959) and has since come to be known as the *pressure solution*. The pressure solution is based on four assumptions (two kinematic and two on the state of stress): (i) points on a vertical line before deformation lie on a parabola after loading (*parabolic bulging*); (ii) horizontal planes remain horizontal; (iii) the stress state is assumed to be dominated by the internal pressure, p (which gives the solution its name), such that the normal stress components are all approximately equal to $-p$; and (iv) the in-plane shear stresses (in the plane parallel to the end supports) are negligible (Kelly and Konstantinidis, 2011). Although it was first used for incompressible material, it was later extended to include bulk compressibility effects. Expressions for the compression stiffness of rubber layers including compressibility have been developed for rubber layers with various geometries: circular

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(Chalhoub and Kelly, 1990), annular (Constantinou et al., 1992), infinite-strip (Chalhoub and Kelly, 1991), square (Koh and Kelly, 1989; Kelly, 1997), rectangular (Koh and Lim, 2001; Kelly and Konstantinidis, 2011).

Various efforts have been made to remove the assumptions of the pressure solution. For instance, Koh and Kelly (1989) used only the two kinematic assumptions of the pressure solution (i.e., removing the normal stress assumption) and a variable transformation method to develop solutions for the compression modulus of a square layer bonded to rigid supports. The same approach was applied by Koh and Lim (2001) to a rectangular layer. Tsai and Lee (1998) proposed an approach that eliminated the normal stress assumption and used *mean* pressure, instead, to derive expressions for infinite-strip, circular and square elastic layers bonded to rigid supports. Tsai (2005) applied this approach to a rectangular layer, developing a single series solution for the compression modulus.

Papoulia and Kelly (1996) followed an approach using the minimum potential energy and Hellinger–Reissner variational principles to estimate the compression modulus of nearly incompressible elastic layers. Pinarbasi et al. (2006) developed an analytical solution based on a modified version of the Galerkin method for the analysis of infinite-strip elastic layers bonded to rigid supports. The order of the theory is based on the number of shape functions considered in the displacement expansions. The method was applied to circular and annular layers in Pinarbasi et al. (2008). The formulations in these studies are applicable to elastic material with a broad range of Poisson's ratio, but they converge to the pressure solution for large values of Poisson's ratio (or large bulk-to-shear modulus ratio), especially for layers with larger shape factor, S (defined as the ratio of the loaded area to the load-free area that is free to bulge) (Papoulia and Kelly, 1996). The pressure solution is considered to provide accurate results for rubber layers with, say, $S > 5$ (Kelly, 1997).

The aforementioned studies developed solutions for the compression modulus of rubber layers under the assumption that the layers are bonded to *rigid* supports. Osgooei et al. (2014) showed that these solutions, developed for single layers, provide accurate estimates of the compression stiffness of a multilayer rubber bearing reinforced with steel shims by treating the layers as springs in series. The development of fiber-reinforced laminated rubber bearings has prompted various investigations on the compressive behavior of rubber layers bonded to axially *flexible* supports, representing the fiber reinforcement. To achieve this, Kelly (1999) proposed a pressure solution approach whereby the assumed displacement field is modified to include the stretch of the reinforcement. The approach was used, assuming incompressible material, to develop solutions for infinite-strip-shaped (Kelly, 1999), circular-shaped (Tsai and Kelly, 2001), and rectangular-shaped (Tsai and Kelly, 2001; 2002) layers. The effect of bulk compressibility was later included to develop solutions for rubber layers bonded to extensible reinforcements for different geometries: infinite strip (Kelly, 2002; Kelly and Takhirov, 2002), rectangular (Angeli et al., 2013; Kelly and Van Engelen, 2015), annular (Pinarbasi and Okay, 2011), and circular (Kelly and Calabrese, 2013). Tsai (2004), (2006) relaxed the normal stress assumption of the pressure solution to develop solutions for infinite strip and circular elastic layers bonded to extensible reinforcement. The compression stiffness of a laminated rubber bearing was estimated by taking into account the fact that layers in the middle portion of the bearing will extend laterally more than those closer to the top and bottom supports through the introduction of an assumed parabolic shape. Pinarbasi and Mengi (2008) extended the approach presented in Pinarbasi et al. (2006) to infinite-strip-shaped elastic layers bonded to extensible reinforcement.

In all these studies, aimed at quantifying the compressive characteristics of rubber layers, it is assumed that the layers are bonded to either rigid or extensible reinforcement. The intent is usually to provide the compression modulus of a layer, which can then be used to compute the compressive stiffness of a laminated steel- or fiber-reinforced rubber bearing. The resulting solutions are appropriate for bearings that are bonded to steel end plates, as is the case almost always for seismic isolators. However, rubber bearings are very commonly used in unbonded applications. Under unbonded boundary conditions, the friction between the rubber and the top and bottom supports of the bearing is responsible for the development of shear stresses under pure compressive load. These surface shear stresses, τ_s, increase outwardly towards the edges of layer, while the pressure, p, decreases. If the rubber–support interfaces are characterized by Coulomb friction with coefficient of friction μ, the surface shears are limited to $\tau_s \leq \mu p$, which means that at some point slip must occur. Although friction in rubber is relatively high, smooth support surfaces or the introduction of some level of lubrication, either intentionally or accidentally, can reduce the frictional restraint along the support–rubber interfaces, resulting in slip and a reduction in the compression modulus. Kelly and Konstantinidis (2009) investigated the effect of slip on the compression properties of a single rubber layer restrained by friction along its top and bottom supports, as well as on a rubber layer bonded to a rigid support on one end (representing a steel shim) but restrained by friction on the other. The study focused on infinite-strip-shaped layers of incompressible rubber and concluded that slip can significantly reduce the compression modulus of the layer. This conclusion has been confirmed by Rastgoo Moghadam and Konstantinidis (2014) using finite element analysis. For unbonded multilayer rubber bearings, especially with only a few layers, this can in turn result in an appreciable reduction in the overall vertical stiffness of the bearing.

Various analytical and experimental studies (Gent and Lindley, 1959; Koh and Kelly, 1989; Kelly and Konstantinidis, 2011) have pointed out that consideration of the bulk compressibility of the elastomer in the compression analysis of rubber layers can have a significant effect on the pressure distribution, the maximum shear strain that is developed by the constraint of the rigid supports on the top and bottom of the bonded rubber layer, and the compression modulus, especially for bearings with large shape factor (Kelly and Konstantinidis, 2011; Van Engelen et al., 2016). This paper presents a theoretical analysis of the compression behavior of strip and circular rubber layers taking into account bulk compressibility and contact slip at the supports. Fig. 1 shows a photograph of a typical thin rubber pad. The analysis presented herein is for a single-layer pad restrained by Coulomb friction at the top and bottom supports, while the compressive behavior of a friction-restrained multilayer bearing with compressible material will be investigated in a future study. Although the description of

Fig. 1. Unbonded rubber pad.