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Normal and tangential stiffnesses of rough surfaces in contact via an imperfect interface model

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ABSTRACT

In this paper a spring-like micromechanical contact model is proposed, aiming to describe the mechanical behavior of two rough surfaces in no-sliding contact under a closure pressure. The contact region between two elastic bodies is described as a thin damaged interphase characterized by the occurrence of non-interacting penny-shaped cracks (*internal* cracks). By combining a homogenization approach and an asymptotic technique, tangential and normal equivalent contact stiffnesses are consistently derived. An analytical description of evolving contact and no-contact areas with respect to the closure pressure is also provided, resulting consistent with theoretical Hertz-based asymptotic predictions and in good agreement with available numerical estimates. Proposed model has been successfully validated through comparisons with some theoretical and experimental results available in literature, as well as with other well-established modeling approaches. Finally, the influence of main model parameters is addressed, proving also the model capability to catch the experimentally-observed dependence of the tangent-to-normal contact stiffness ratio on the closure pressure.

1. Introduction

Analytical and numerical modeling of contact problems related to rough surfaces can be surely considered as an open and challenging research topic, strictly associated to many applications in different engineering fields. From a computational point of view, it is possible to identify a class of modeling problems in which it is neither possible nor convenient to account for a fine and detailed description of the contact regions, although local contact features may strongly affect the overall mechanical response for the problem under investigation. In these cases, a possible strategy is based on modeling contact scenarios by introducing suitable stiffness and dashpot distributions at the contact nominal interface, allowing to upscale at the macroscale the influence of dominant contact mechanisms occurring at the roughness scale. In this context and as reviewed by Baltazar et al. (2002), starting from fundamental results of classic contact theories and accounting for main microgeometric features at the contact interface, several theoretical and numerical models have been proposed in the specialized literature (namely, spring-like models), aiming to consistently derive some equivalent stiffnesses.

One of the earliest contact model for elastic rough surfaces was proposed by Greenwood and Williamson (1966). This model is based on the Hertz contact solution for curved elastic nominally-flat surfaces (Mindlin, 1949) and it accounts for a statistical distribution of non-interacting asperities. Moreover, Yoshioka and Scholz (1989) proposed an elastic contact model via a statistical approach that allows to describe possible oblique contact conditions among asperities. By combining the Hertz–Mindlin theory (Mindlin, 1949) and the previously-introduced Greenwood-Williamson contact model, Sherif and Kossa (1991) and Krolikowski and Szczepek (1993) provided an analytical description of normal and tangential contact stiffnesses, in order to establish a theoretical interpretation for the experimental results they obtained. In this case, the contact between two nominally-flat rough surfaces is modeled as the contact between two elastic surfaces, one of which is ideally flat and the other is nominally flat but covered with many spherically-shaped asperities. A generalization of such an approach was developed by Baltazar et al. (2002), accounting also for a possible contact misalignment. Nevertheless, a possible common drawback of all the aforementioned contact models is that they are generally based on a stochastic approach. Accordingly, in order to

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make them practically applicable, the identification of a number of statistic parameters, often not easily estimable (McCool, 1986), is required.

A crucial aspect in deriving reliable contact solutions is related to the description of the contact area and its evolution with respect to the closure pressure (Johnson, 1987). Starting from the analytical solution of Westergaard (1939), Johnson et al. (1985) developed a model for the elastic contact between a two-dimensional wavy surface and a rigid flat plane, proposing an analytic description of the contact area in the asymptotic limit cases of early contact (namely, for small values of the closure pressure) and of nearly-full contact conditions (high values of the closure pressure). More recently, Yastrebov et al. (2014) proposed a refined numerical approach consisting in a FFT-based boundary-element formulation, and they obtained an accurate numerical description of the contact-area evolution with the closure pressure in the case of the elastic contact between a wavy surface and a flat plane.

Several experimental studies can be found in the literature addressing the mechanical behavior of rough surfaces in no-sliding contact under closure-pressure conditions (e.g., Krolikowski et al., 1989; Sherif and Kossa, 1991; Krolikowski and Szczepek, 1993; Baltazar et al., 2002; Dwyer-Joyce and Gonzalez-Valadez, 2003; Gonzalez-Valadez et al., 2010), providing also estimates for normal and tangential contact stiffnesses. For instance, Krolikowski and coworkers (Krolikowski and Szczepek, 1993; Krolikowski et al., 1989) proposed contact-stiffness measures through an ultrasonic method, based on the measurement of the reflection coefficient of ultrasonic waves at the contact interface. Sherif and Kossa (1991) employed an experimental technique based on the evaluation of the local natural frequencies at the contact region. Gonzalez-Valadez et al. (2010) proposed results based on ultrasonic tests and accounting also for loading-unloading cycles of the closure pressure. As a matter of fact, experimental results confirm that: high stress concentrations appear at the contact region, and they result practically unaffected by the shape of bodies in contact at a suitable distance from the contact area (Johnson, 1987; Johnson et al., 1985); hysteresis phenomena occur at the interface (as a result of the plastic deformation localized at the asperity tips) in the case of cycling loads (Gonzalez-Valadez et al., 2010); null values of interface stiffnesses are achieved when the closure pressure tends to zero (Gonzalez-Valadez et al., 2010).

In this paper a novel spring-like theoretical model for no-sliding contact under a closure pressure is proposed. Incremental normal and tangential equivalent stiffnesses at the nominal contact interface are derived, by assuming contact microgeometry be described by isolated *internal* cracks (Sevostianov and Kachanov, 2008a; 2008b) occurring in a thin interphase region. In detail, effective mechanical properties at the contact zone are consistently derived following the imperfect interface approach recently adopted by Lebon and coworkers (Fouchal et al., 2014; Rekik and Lebon, 2010; 2012), by coupling a homogenization approach for microcracked media based on the non-interacting approximation (Kachanov, 1994; Kachanov and Sevostianov, 2005; Sevostianov and Kachanov, 2013; Tsukrov and Kachanov, 2000) and the matched asymptotic expansion method, introduced by Sanchez-Palencia (1987) and Sanchez-Palencia and Sanchez-Hubert (1992) and recently employed by Lebon and Rizzoni (2011), Rizzoni and Lebon (2013), Rizzoni et al. (2014) and Lebon and Zaittouni (2010).

The proposed model is detailed in Section 2, and its validation is provided by comparing numerical results with available theoretical and experimental findings (Section 3.1). Model effectiveness is also proved for a wide range of closure-pressure values by comparing proposed results with those obtained via the contact model introduced by Sherif and Kossa (1991) (Section 3.2). Afterwards, the influence of main model parameters is investigated in Section 3.3, and finally some conclusions are traced in Section 4.

2. Contact model

2.1. General framework

Let the contact problem $\mathcal{C}$ be introduced by considering two continuous bodies Ω_1 and Ω_2, comprising linearly-elastic isotropic materials (E_i and ν_i, with $i = 1, 2$, being Young modulus and Poisson ratio, respectively), in no-sliding contact via non-conforming rough surfaces under a closure pressure condition (Fig. 1). Let $\mathcal{S} \subset \mathbb{R}^2$ be the nominal contact interface, belonging to the interface plane π. Let a Cartesian frame $(O, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ be introduced, with x_1, x_2 and x_3 the corresponding coordinates. The origin O belongs to π, and $\mathbf{e}_3$ is orthogonal to π and directed outward from Ω_2.

Normal and tangential incremental contact stiffnesses ($K_N^{\mathcal{C}}$ and $K_T^{\mathcal{C}}$, respectively) per unit nominal contact area in $\mathcal{S}$ are defined as:

$$K_N^{\mathcal{C}} = \frac{\mathrm{d}\mathcal{F}_N}{\mathrm{d}w}, \quad K_T^{\mathcal{C}} = \frac{\mathrm{d}\mathcal{F}_T}{\mathrm{d}s} \tag{1}$$

where dw and ds are the increments of the relative displacements at the contact interface region in normal (i.e., along $\mathbf{e}_3$) and tangential (i.e., parallel to π) directions, and $\mathrm{d}\mathcal{F}_N$ and $\mathrm{d}\mathcal{F}_T$ are the increments of the normal and tangential forces transmitted through the unit contact area, respectively. Contact microgeometry is assumed to be isotropic in $\mathcal{S}$ and thereby the tangential contact stiffness $K_T^{\mathcal{C}}$ can be postulated as independent from the tangential direction.

In agreement with the approach adopted by Westergaard (1939) and by Johnson et al. (1985), contact microgeometry is modeled by describing no-contact regions as parallel penny-shaped *internal* cracks (Sevostianov and Kachanov, 2008a; 2008b) lying on the interface plane π. Coplanar mechanical interactions among cracks are considered negligible, resulting in the non-interacting approximation (Kachanov, 1994; Sevostianov and Kachanov, 2013). Accordingly, the region close to the nominal contact interface $\mathcal{S}$ is regarded as an imperfect interphase $\mathcal{B}^\varepsilon$, defined as the thin layer having $\mathcal{S}$ as the middle section and ε as the uniform small thickness, and weakened by identical and parallel penny-shaped cracks of radius b (Fig. 1).

Referring to a simplistic idealization of the contacting rough surfaces via bi-sinusoidal wavy-like smooth surfaces, both of them with wavelength λ and amplitude Δ (such that $\Delta \ll \lambda$), a ε-thick representative elementary volume (REV) at the contact interface, and occupying the region $\Delta\Omega^\varepsilon \subset \mathcal{B}^\varepsilon$, can be conveniently introduced as sketched in Fig. 1.

Accordingly, the contact problem $\mathcal{C}$ is faced by introducing an auxiliary model problem $\mathcal{A}$, defined on the microcracked interphase $\mathcal{B}^\varepsilon$ and described via the REV.

2.2. Imperfect interface approach

Referring to the auxiliary model problem $\mathcal{A}$, and as a notation rule, the following symbols will be adopted: $\Omega_+^\varepsilon = \Omega_1 \setminus \mathcal{B}^\varepsilon$ and $\Omega_-^\varepsilon = \Omega_2 \setminus \mathcal{B}^\varepsilon$, with $\Omega_\pm^\varepsilon$ also referred to as adherents; $\mathcal{S}_\pm^\varepsilon = \Omega_\pm^\varepsilon \cap \mathcal{B}^\varepsilon$ identifying the plane interfaces parallel to π between interphase and adherents. It is assumed that $\Omega_\pm^\varepsilon$ and $\mathcal{B}^\varepsilon$ are perfectly bonded, so that displacement and stress vector fields are ensured to be continuous across $\mathcal{S}_\pm^\varepsilon$.

2.2.1. Homogenization of the microcracked interphase

Let $\Gamma \subset \mathcal{S}$ be the crack middle surface for a penny-shaped crack in $\mathcal{B}^\varepsilon$, and let $\mathbf{u}^+$ and $\mathbf{u}^-$ be the displacement vectors at the parallel-to-$\mathcal{S}$ crack boundaries. Denote also with $\mathbf{u}_{\mathrm{cod}} = \int_\Gamma (\mathbf{u}^+ - \mathbf{u}^-)\mathrm{d}\Gamma/|\Gamma|$ the average measure of the displacement jump across the crack, in the following referred to as the crack opening displacement vector. In agreement with a well-established