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A new hyperelastic model for anisotropic hyperelastic materials with one fiber family

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ABSTRACT

The main goal of this study is to propose a practical application of a new family of transverse anisotropic invariants by designing a strain energy function (SEF) for incompressible fiber-reinforced materials. In order to validate the usability and creativeness of the proposed model, two different fiber-reinforced rubber materials under uniaxial and shear testing are considered. For each kind of material, numerical simulations based on the proposed model are consistent with experimental results and provide information about the effect of the new family of invariants in the construction of the SEF.

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1. Introduction

These past twenty years, many strain energy functions have been proposed for transversely isotropic materials to investigate the mechanical behavior of biological soft tissues. Usually, these materials are considered as anisotropic due to the collagen fiber behavior Gasser et al. (2006). The number of fiber families is set to 1 to model tissues such as ligament or tendons while it is set to 2 to represent the arterial wall Peyraut et al. (2010). Several constitutive finite element models were built for biological soft tissues, such as ligament and tendons Almeida and Spilker (1998); Weiss et al. (1996). Based on the neo-Hookean model, biomechanical behavior of the arterial wall and its numerical characterization are analyzed and discussed in Holzapfel and Weizsäcker (1998). Holzapfel et al. (2000) introduced structural SEFs for describing the soft biological tissues such as the arterial wall. Based on this model, Zulliger et al. (2004) proposed a SEF for arteries that account for the wall composition and structure.

In general, it is assumed that the mechanical behavior of the material is not affected if the fibers are in a compressive state Holzapfel et al. (2004); Merodio and Ogden (2003). Taking advantage of this situation, most of the papers published in the literature propose to separate the energy density into an isotropic part and an anisotropic part. The first part accounts for the low strain behavior of the ground matrix and the second part captures the behavior of the fibers at higher strain Balzani et al. (2006); Weiss et al. (1996). More recently, an original approach mixing the isotropic and the anisotropic parts in a single SEF was introduced by Ta et al. (2014, 2013). This approach was inspired by the pioneer work of Thionnet and Martin (2006) and is mathematically justified by the theory of invariant polynomials. It provides an alternative to the classical method found in the literature for building invariants and allows to exhibit an integrity basis made of six invariants, some of them being original, in the case of a one-fiber family material. In the same spirit, some complementary results are demonstrated in this paper: (i) one of the six invariants exhibited in Ta et al. (2014) can be excluded from the integrity basis by adding the appropriate transformation in the material symmetry group; (ii) three of the six invariants are well known polyconvex functions; (iii) the last two invariants are original, physically motivated and directly connected to shear effects. Additionally, these two invariants shed a light on the classical mixed invariant $J_5 = Tr(\boldsymbol{C}^2\boldsymbol{M})$ and allows to link it with shear strain while it is often reported in the literature the difficulty to provide a physically-based motivation for J_5. However, up to now and to the best of our knowledge, the mathematical foundations introduced in Ta et al. (2014) have not met a practical extension. The new strain energy function proposed in this paper by using the integrity basis made of the six invariants exhibited in Ta et al. (2014) constitutes a first attempt in this direction. To assess the appropriateness of this new density, numerical simulations are compared with experimental results extracted from the paper published by Ciarletta et al. (2011). For our purpose the interest of the work of Ciarletta et al. is threefold:

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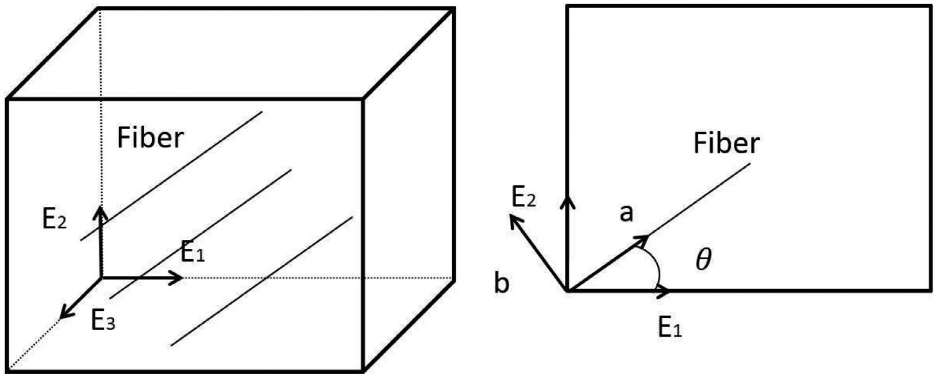

Fig. 1. A fiber-reinforced material with one fiber family.

- It provides a large variety of experimental results by testing two different materials, each in four different situations (tensile and simple shear loadings parallel and transverse to the rubber-reinforcement direction), covering a large scope of the material behavior. It therefore constitutes a good trial for the assessment of models because a single set of material parameters should have to match all the four experimental tests.
- If tensile tests prevail in the literature, shear tests are uncommon although they can be considered as a severe benchmark case for rubber material models. As outlined by Horgan and Murphy (2010): "*The classical problem of simple shear in nonlinear elasticity has played an important role as a basic pilot problem involving a homogeneous deformation that is rich enough to illustrate several key features of the nonlinear theory, most notably the presence of normal stress effects.* $(\cdots)$ *Since shearing is one of the dominant modes of behavior of rubbers in applications, this raises major concerns. Put another way, simple shear is not so simple after all*".
- A new hyperelastic model using a non classical measure of strain is also proposed in Ciarletta et al. (2011). In the same vein, Fereidoonnezhad et al. (2013) have built later a model using this kind of strain, reporting the nonlinearity aspect from the form of the SEF to the strain invariant, and have used the experimental data provided by Ciarletta et al. to assess their model. Our new model can therefore be compared not only with experimental results but also with numerical simulations.

As mentioned before, the model proposed in this paper involves a combination of the six new invariants determined by Ta et al. (2014). This choice is motivated by the fact that these invariants do not require a separation of the SEF into an isotropic and an anisotropic part. Another motivation is the rigorous mathematical foundations used by Ta *et al.* to define those invariants. In the same spirit as the Mooney–Rivlin models in the framework of isotropic hyperelasticity Mooney (1940); Rivlin (1948), we have introduced an original SEF as a polynomial function of these new invariants. The conclusions are as follows:

- A linear or a quadratic expansion of the invariants is not sufficient to well describe the material behavior with the four experimental set-up considered, particularly with the shear test. In fact we prove that any polynomial SEF in the invariants will not be suitable to fit the experimental data.
- A quadratic expansion of the invariants combined with an appropriate power-law form provides accurate predictions of all the experimental results.
- The three invariants K_1, K_3 and K_6 play a distinguished role to model shear effect.

Notations

A bold-face Latin lowercase letter, say $\boldsymbol{a}$, and a bold-face Latin capital letter, say $\boldsymbol{A}$, will denote a vector and second-order tensor, respectively. The standard Euclidean scalar product is symbolized by a double bracket

$$\langle \boldsymbol{C}\boldsymbol{a}, \boldsymbol{a}\rangle = \sum_{i=1}^{3} C_{ij} a_j a_i$$

and the related Euclidean norm is noted $\|.\|$:

$$\|\boldsymbol{u}\| = \langle \boldsymbol{u}, \boldsymbol{u}\rangle^{\frac{1}{2}}$$

The tensor product between two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined by

$$(a \otimes b)_{ij} = a_i b_j$$

2. Preliminaries

In this paper we focus on a fiber-reinforced material with one fiber family of direction $\boldsymbol{a}$ as depicted on Fig. 1. We assume that $\boldsymbol{a}$ lies in the plane $(\boldsymbol{E}_1, \boldsymbol{E}_2)$ and forms an angle θ with $\boldsymbol{E}_1$

$$\boldsymbol{a} = \begin{pmatrix} c \\ s \\ 0 \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} -s \\ c \\ 0 \end{pmatrix} \text{ with } c = \cos(\theta), \quad s = \sin(\theta) \tag{1}$$

Practically, we will only consider the following two cases where the fibers are parallel ($\theta = 0$) or transverse ($\theta = \frac{\pi}{2}$) to $\boldsymbol{E}_1$

$$\text{Parallel: } \boldsymbol{a} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{c} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix};$$

$$\text{Transverse: } \boldsymbol{a} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, \quad \boldsymbol{c} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2}$$

The group of all orthogonal transformations of $\mathbb{R}^3$ is denoted by $\mathcal{Q}^3 = \{\boldsymbol{Q} \in \mathcal{M}_{3\times3}(\mathbb{R}), \boldsymbol{Q}\boldsymbol{Q}^{\boldsymbol{T}} = \boldsymbol{Q}^{\boldsymbol{T}}\boldsymbol{Q} = \boldsymbol{I}\}$, where $\boldsymbol{I}$ is the identity matrix and $\mathcal{M}_{3\times3}(\mathbb{R})$ the set of 3×3 real matrices. We denote by $\mathcal{G}$ the material symmetry group containing all the orthogonal transformations of $\mathcal{Q}^3$ leaving invariant the material structure. This group $\mathcal{G}$ can be described as the group of all rotations around the fiber direction $\boldsymbol{a}$. Using a mathematical argument based on an extension of the Reynolds operator, in order to account for the infinite cardinality of $\mathcal{G}$, Ta et al. (2014) have demonstrated that the following six invariant polynomials form an integrity basis of the ring of invariant polynomials under the action of $\mathcal{G}$

$$\begin{aligned} &K_1 = \rho_1; \quad K_2 = \rho_2 + \rho_3; \quad K_3 = \rho_5^2 + \rho_4^2; \quad K_4 = \rho_6^2 - \rho_2\rho_3 \\ &K_5 = (\rho_5^2 - \rho_4^2)\rho_6 + \rho_4\rho_5(\rho_2 - \rho_3); \quad K_6 = (\rho_4^2 - \rho_5^2)(\rho_2 - \rho_3) \\ &\quad + 4\rho_4\rho_5\rho_6 \end{aligned} \tag{3}$$

where the ρ_i stand for

$$\begin{aligned} &\rho_1 = \langle \boldsymbol{C}\boldsymbol{a}, \boldsymbol{a}\rangle; \quad \rho_2 = \langle \boldsymbol{C}\boldsymbol{b}, \boldsymbol{b}\rangle; \quad \rho_3 = \langle \boldsymbol{C}\boldsymbol{c}, \boldsymbol{c}\rangle \\ &\rho_4 = \langle \boldsymbol{C}\boldsymbol{a}, \boldsymbol{b}\rangle; \quad \rho_5 = \langle \boldsymbol{C}\boldsymbol{a}, \boldsymbol{c}\rangle; \quad \rho_6 = \langle \boldsymbol{C}\boldsymbol{b}, \boldsymbol{c}\rangle \end{aligned} \tag{4}$$

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