International Journal of Solids and Structures 84 (2016) 17–26

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Nonlinear frequency shift behavior of graphene–elastic–piezoelectric laminated films as a nano-mass detector

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ARTICLE INFO

Article history:
Received 26 April 2015
Revised 2 December 2015
Available online 23 January 2016

Keywords:
Nano-mass detector
Graphene
Piezoelectric
Nonlinear resonant
Scale effect

ABSTRACT

This paper presents an analytical method to investigate the nonlinear frequency shift of graphene–elastic–piezoelectric (GEP) laminated films as a resonant mass detector. Based on the nonlocal elastic theory and nonlinear geometrical relation, the nonlinear dynamic governing equations containing deflection function, stress function and electric potential function are constructed by Hamilton's principles, then are solved by Galerkin method and the iterative homotopy harmonic balance method. The effects of some key parameters on the nonlinear frequency shift are described. Results show that both nonlinearity and nonlocal parameter appear in significant influences on the frequency shift of GEP laminated nano-mass detector, and the frequency shift can be controlled by adjusting the external voltage acted on the piezoelectric layer when different mass particles attach to the detector surface. The present work can serve as a guideline for the design of a nanoscale resonant mass detector or other GEP-films based electromechanical resonator sensors.

1. Introduction

Over the past years, sensors are increasingly used in engineering applications, such as structural monitoring, food or medicine detection, environmental monitoring, biomedical science, chemical process, etc. (Wang and Arash, 2014; Kang et al., 2012; Jalali and Naei, 2014; Joshi et al., 2011). The emerging of nanotechnology has promoted the miniaturization of sensors to nanoscale, which can satisfy the request for extremely sensitivity performance, high robustness and more integrating functions packaged in a small elecromechanical element. Among kinds of sensors, nano-mass detector as a crucial device has attracted tremendous attention because of the increasing demand in life and biomedical sciences (Hood et al., 2004; Yan et al., 2013;Benz et al., 2015; Barnard et al., 2012).

An outer atom or molecule attached on the surface of a resonant mass detector can be detected by capturing the shift of its resonant frequencies or wave velocities (Lavrik and Datskos, 2003; Ekinci et al., 2004; Chaste et al., 2012). Among various nanomaterials used in the design of mass detectors, the potentials of carbon nano-materials represented by carbon nanotubes (CNTs) and graphene have been investigated by experiments and theories owing to their excellent mechanical, electrical, thermal properties (Iijima, 1991; Novoselov et al., 2004, 2005; Geim and Novoselov, 2007). As to CNTs, because of carbon particle could reduce the resonant frequency of CNTs (Wang et al., 2000), Mateiu et al. (2005) introduced an approach to use multi-walled CNTs to measure mass (10^{-12}–10^{-15} g) through resonant frequency shift. In another work (Lassagne et al., 2008), a single-walled CNTs was used to sense ultra-sensitive mass with a resolution of 25 zep to gram (25×10^{-21} g). As to graphene, based on nonlocal Kirchhoff plate theory, Shen et al. (2012) studied simply supported single-layered graphene as mass sensor according to the frequency shift from different mass attachment. Bunch et al. (2007) fabricated nanoelectromechanical systems with single-layered and multilayered graphene to investigate their applications in sensing mass, force and charge. Using molecular structural mechanics, Sakhaee-Pour et al. (2008) reported the potential of single-layered graphene sheets in the design of mass sensors and atomistic dust detectors. Then by molecular dynamics simulations, Arash et al. (2011) showed that a graphene based resonator sensor could be applied in the detection of gas molecule when a noble gas attached to graphene sheet at random position. Though a plethora of works have focused on the fabrications and applications of carbon nanomaterial based mass sensors or detectors, the predictable natural frequency of a given nano-material may restrict the applications in some extreme circumstances, such as high temperature, electric field, pressure, etc. Hence, layer structures having a tunable resonant frequency should be introduced to make up this defect.

Because grapheme appears in high electron mobility and large absorption surface (Schedin et al., 2007), the graphene on the top of laminated films may enhance the adsorbent sensitivity

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http://dx.doi.org/10.1016/j.ijsolstr.2015.12.011

of graphene composites based sensors. Considering the unique electro-mechanical conversion capability of piezoelectric material as a smart material used in sensors and actuators (Zhang et al., 2011), it can be envisaged that a laminated films composed of graphene, piezoelectric (ZnO, PVDF, PZT, $LiNbO_3$, etc.) and other kinds of materials may be a good candidate of nano-sensors, which can make fully use of their distinctive properties. Recently, the laminated films have been successfully synthesized. Shin et al. (2011) fabricated a loud speaker with a layer PVDF and two layers graphene at the up and bottom, respectively. And a similar investigation was executed by Xu et al. (2013). Utilizing the pulsed laser deposited technology, Zeng et al. (2014) studied the mechanism of laminated ZnO/graphene films by growing ZnO films on graphene layers and showed the potential in the fabrication of flexible optoelectronic devices. Moreover, Battista et al. (2012) demonstrated a novel scheme to improve the absorption of solar radiation by coating $LiNbO_3$ on graphene in a solar energy harvesting system. Thereby, it is foreseeable that graphene–piezoelectric laminated structures will have broad applications in nano-engineering.

Since controlled nanoscale experiments on graphene–piezoelectric based materials are extremely difficult, this paper presents an analytical method to investigate the nonlinear frequency shift of a nano-mass detector composed of the graphene–elastic–piezoelectric laminated films. Firstly, the nonlinear dynamic governing equations containing deflection, stress and electric potential functions are derived based on Hamilton's principles, nonlocal elastic theory containing the length force among atoms and von Karman geometrical relation, then are solved by Galerkin method and the homotopy harmonic balance method. Results show that the nonlinearity behavior of GEP laminated films degenerates the sensitivity of nano-mass detector, and the effect of nonlinearity behavior on the sensitivity of nano-mass detector increases with the increase of attached mass; the external voltage applied on the piezoelectric player in GEP-based mass detector appears in a larger effect on the frequency shift, adjusting the external voltage on the piezoelectric player can enhance the sensitivity of GEP-based mass detector; the small scale effect makes the influence of attached mass on the frequency shift Δf_{NL} of GEP-based mass detector decrease. These meaningful results may be helpful for the application of ultra-sensitive mass detecting and will stimulate further interest in this topic.

2. Nonlinear governing equations of nano-mass detection sensors

The schematic of sensor system described by a graphene–elastic–piezoelectric GEP laminated films attached by particles (such as biomolecules, viruses, buckyballs, etc.) is shown in Fig. 1, where the single-layered graphene is deposited on the upper surface of elastic layer while the piezoelectric layer is fabricated at the bottom.

The attached mass is considered as a concentrated mass on the surface of GEP films, thereby, the transverse load per unit area, induced by attached multiple nanoparticles with random positions (Asemi et al., 2015), is written as

$$q = -\sum_{k=1}^{N} m_k \delta(x - x_k)\delta(y - y_k)\frac{\partial^2 w}{\partial t^2} \tag{1}$$

where m_k is the mass of the kth particle attached at position (x_k,y_k), Nrepresents the total attached particle number, and δ is the Dirac delta function defined by

$$\delta(x - x_k) = \begin{cases} \infty & x = x_k \\ 0 & x \neq x_k \end{cases} \tag{2}$$

Here, the geometric nonlinearity is considered to study the vibration characteristics of mass detector, where because the deflection of the GEP laminated films is much larger than in-plane displacements, the in-plane displacements is negligible (Raju et al., 1976) in the nonlinear vibration sensing process. Utilizing Hamilton's principles and integrating, and setting the coefficient of δu, δv, δw and $\delta\phi$ to zero, the nonlinear vibration governing equations for this mass detector system are given by

$$\delta u : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\frac{\partial^2 u}{\partial t^2} - I_1\frac{\partial^3 w}{\partial x \partial t^2} \tag{3a}$$

$$\delta v : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\frac{\partial^2 v}{\partial t^2} - I_1\frac{\partial^3 w}{\partial y \partial t^2} \tag{3b}$$

$$\begin{aligned}\delta w \ : \ & \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial}{\partial x}\left(N_x\frac{\partial w}{\partial x} + N_{xy}\frac{\partial w}{\partial y}\right) \\ & + \frac{\partial}{\partial y}\left(N_{xy}\frac{\partial w}{\partial x} + N_y\frac{\partial w}{\partial y}\right) + q \\ & = I_0\frac{\partial^2 w}{\partial t^2} + I_1\left(\frac{\partial^3 u}{\partial x \partial t^2} + \frac{\partial^3 v}{\partial y \partial t^2}\right) - I_2\left(\frac{\partial^4 w}{\partial x^2 \partial t^2} + \frac{\partial^4 w}{\partial y^2 \partial t^2}\right)\end{aligned} \tag{3c}$$

$$\begin{aligned}\delta\phi \ : \ & \int_{z_0}^{z_1}\left[\cos\left(\frac{\pi\,(z + (h_2 + h_3)/2)}{h_1}\right)\frac{\partial D_x}{\partial x}\right. \\ & + \cos\left(\frac{\pi\,(z + (h_2 + h_3)/2)}{h_1}\right)\frac{\partial D_y}{\partial y} \\ & \left. + \frac{\pi}{h_1}\sin\left(\frac{\pi\,(z + (h_2 + h_3)/2)}{h_1}\right)D_z\right]dz = 0\end{aligned} \tag{3d}$$

where

$$(I_0, I_1, I_2) = \sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\rho_k(1, z, z^2)dz \tag{4}$$

and u, v, and w are the components of displacements in the x, y and z directions, respectively. In view of the large amplitude motion of GEP films, the Von Kármán nonlinear geometric relations are given by

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2} = \varepsilon_x^0 + zK_x \tag{5a}$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\frac{\partial^2 w}{\partial y^2} = \varepsilon_y^0 + zK_y \tag{5b}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} - 2z\frac{\partial^2 w}{\partial x \partial y} = \gamma_{xy}^0 + zK_{xy} \tag{5c}$$

where

$$K_x = -\frac{\partial^2 w}{\partial x^2},\ K_y = -\frac{\partial^2 w}{\partial y^2} \text{ and } K_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}.$$

Presently, there exist two kinds of nonlocal models: the nonlocal weakening model and the nonlocal strengthening model, but both models are proved to be valid (Li, 2014a, 2014b). Based on the nonlocal elastic theory containing the long-range forces among atoms, the stress–strain relations of piezoelectric layer are written as

$$\begin{aligned}(1 - \mu^2\nabla^2)\begin{Bmatrix}\sigma_x \\ \sigma_y \\ \sigma_{xy}\end{Bmatrix} = & \begin{bmatrix}C_{11}^p & C_{12}^p & 0 \\ C_{12}^p & C_{22}^p & 0 \\ 0 & 0 & C_{66}^p\end{bmatrix}\begin{Bmatrix}\varepsilon_x \\ \varepsilon_y \\ \gamma_{xy}\end{Bmatrix} \\ & - \begin{bmatrix}0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}E_x \\ E_y \\ E_z\end{Bmatrix}\end{aligned} \tag{6}$$