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Automatic rationalization of yield-line patterns identified using discontinuity layout optimization

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ABSTRACT

The well-known yield-line analysis procedure for slabs has recently been systematically automated, enabling the critical yield-line pattern to be identified quickly and easily, whatever the slab geometry. This has been achieved by using the discontinuity layout optimization (DLO) procedure, which involves using optimization to identify the critical layout of yield-line discontinuities interconnecting regularly spaced nodes distributed across a slab. However, whilst highly accurate solutions can be obtained, the corresponding yield-line patterns are often quite complex in form, especially when relatively dense nodal grids are employed. Here a method of rationalizing the DLO-derived yield-line patterns via a geometry optimization post-processing step is described. Geometry optimization involves adjusting the positions of the nodes, thereby simultaneously simplifying and improving the accuracy of the solution. The mathematical expressions involved are derived analytically, and various practical issues are highlighted and addressed. Finally, an interior point optimizer is used to obtain rationalized solutions for a variety of sample slab analysis problems, clearly demonstrating the efficacy of the proposed rationalization technique.

1. Introduction

The yield-line method of analysis proposed by Johansen (1943) provides a powerful means of computing the collapse load of a reinforced concrete slab. The method, which provides upper bound solutions within the context of the formal theorems of plasticity, requires a kinematically admissible failure mechanism to be prescribed, defined by means of a yield-line pattern. The early focus was on slabs with relatively simple geometries (e.g., Johansen, 1943, 1968) because, at the time, systematic means of identifying the critical failure mechanism for irregularly shaped slabs were not available. Subsequently Chan (1972) and Munro and Da Fonseca (1978) proposed a means of automatically identifying the critical yield-line pattern. This involved discretizing a slab using rigid finite-elements, with the critical yield-line pattern then obtained automatically via linear optimization. However, because yield-lines were restricted to forming only at the edges of the finite-elements, the resulting yield-line patterns were significantly influenced by the initial mesh topology. Attempting to address this issue, various workers proposed the use of 'geometry optimization' to subsequently adjust the positions of selected nodes in a post-processing phase. For example, Johnson (1994, 1995) proposed that this be achieved via the use of sequential linear programming. Other workers to propose a similar approach included Thavalingam et al. (1999), who employed a conjugate gradient optimizer, and Ramsay and Johnson (1997, 1998), who used a direct search solver. However, as indicated by Ramsay et al. (2015), the outcomes will be affected by the initial mesh topology, and a poor initial solution will render any subsequent geometry optimization phase largely ineffective. Another issue is the need to manually identify yield-lines from the finite-element meshes; any misinterpretation can reduce the efficacy of the geometry optimization phase. This has been described as being 'difficult' (e.g., Johnson, 1994, Thavalingam et al., 1999). As an alternative, plate formulations in which deformations can take place within elements, rather than just at element boundaries, have been proposed, with pioneering work in this field undertaken by Hodge and Belytschko (1968) and Anderheggen and Knöpfel (1972). However, with such formulations the yield-line pattern can be somewhat difficult to discern.

More recently, Jackson (2010) and Jackson and Middleton (2013) used a lower-bound finite element solution to derive 'yield-line indicators', which could be used to infer the likely general form of a critical yield line pattern. This then enabled a more refined yield-line pattern to be identified via a geometry optimization step. The resulting procedure allowed reasonable yield-line analysis solutions to be obtained for complex slab problems. However, as the procedure involved a manual interpretation step, a truly systematic means of automatically identifying the critical yield-line pattern remained to be found.

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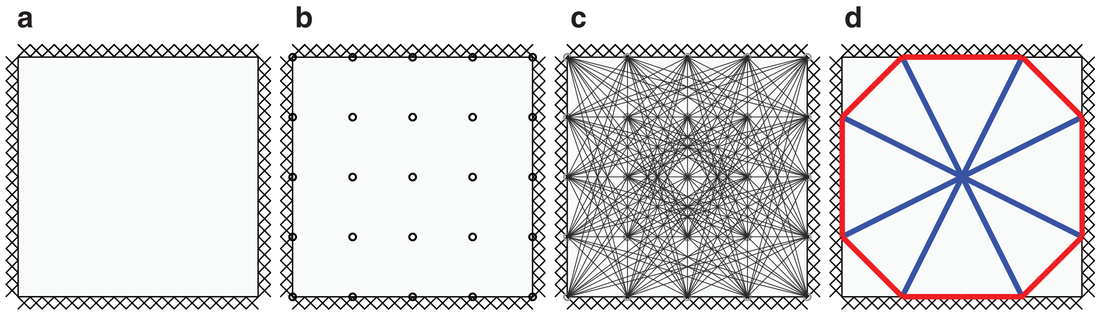

Fig. 1. Steps in the DLO procedure: (a) define slab geometry and properties; (b) discretize slab using nodes; (c) interconnect nodes with potential yield-lines; (d) use optimization to identify optimal subset of yield-lines, and resulting yield-line pattern .

Recently, this goal was achieved by Gilbert et al. (2014), who used discontinuity layout optimization (DLO) to automate the process of identifying the most critical yield-line pattern. Instead of discretizing the problem using elements arranged in a finite element mesh, when using DLO the slab area is populated by nodes, and these are then interconnected with a large set of potential yield-lines, which are free to cross-over one another. A highly efficient optimization process is then used to find the critical subset of yield-lines involved in the critical failure mechanism. An overview of the steps involved in the DLO procedure is shown in Fig. 1. Improved solutions can be obtained by using an increased number of nodes; the resulting increased number of potential yield-lines can be handled efficiently using the adaptive solution scheme proposed for truss layout optimization by Gilbert and Tyas, 2003, and used for this application in Gilbert et al., 2014. However, whilst highly accurate solutions can be obtained using the DLO procedure, the corresponding yield-line patterns are often quite complex in form, especially when relatively dense nodal grids are employed. In an attempt to address this, a modified formulation was also proposed by Gilbert et al. (2014). The modified formulation involved penalizing short yield-lines, leading to solutions that were generally simpler in form than the original. However, these solutions were also less accurate (i.e. the gap between the exact and numerical solution was increased). In the present paper a geometry optimization step will instead be used to rationalize the yield-line patterns, with a view to simultaneously simplifying the yield-line patterns and improving the solutions (i.e. so that the gap between the exact and numerical solution reduces).

The proposed procedure clearly has similarities with the procedure put forward by Johnson (1994, 1995), which also involved the use of a geometry optimization step. However, in the proposed procedure the rationalization process starts from a yield-line pattern obtained using DLO, which is a much better starting point than a yield-line pattern derived from a rigid finite element analysis. Also, here the relevant geometry optimization formulae will be derived analytically, thus permitting a wider variety of optimization methods to be applied. These distinguishing features can be expected to ensure that performance is much improved. Note also that the proposed procedure is similar to the procedure recently proposed for rationalizing trusses identified using layout optimization (He and Gilbert, 2015); also the use of a geometry optimization step to improve very coarse resolution DLO solutions has recently been proposed for in-plane analysis problems by Bauer and Lackner (2015).

The paper is organized as follows: (i) the new DLO-based automated yield-line analysis procedure is first introduced; (ii) the geometry optimization problem is defined and relevant mathematical expressions are given; (iii) implementation issues are considered and addressed; (iv) various numerical examples are used to demonstrate the efficacy of the procedure; (v) conclusions from the study are presented.

2. Automated yield-line analysis using DLO

2.1. Overall problem formulation

The kinematic DLO limit analysis formulation for a weightless slab can be written as an optimization problem as follows (after Gilbert et al., 2014):

$$\min_{d,p} \quad \lambda \mathbf{f}_L^T \mathbf{d} = \mathbf{g}^T \mathbf{p} \tag{1a}$$

$$\text{s.t.} \quad \mathbf{B}\mathbf{d} = \mathbf{0} \tag{1b}$$

$$\mathbf{N}\mathbf{p} - \mathbf{d} = \mathbf{0} \tag{1c}$$

$$\mathbf{f}_L^T \mathbf{d} = 1 \tag{1d}$$

$$\mathbf{p} \geq \mathbf{0}, \tag{1e}$$

where the objective is to minimize the internal work done along yield-lines (1a), subject to compatibility at nodes (1b), plastic flow requirements (1c), a unit displacement constraint, defined according to the principle of virtual work, (1d), and a constraint that ensures that the internal work done must be positive (1e). And where λ is a dimensionless load factor, and **p** and **g** are vectors containing plastic multipliers and their corresponding work equation coefficients. Also **B** is a suitable compatibility matrix containing direction cosines for the yield-lines, and **d** contains relative displacements along yield-lines, as shown in Fig. 2 (where θ_n, θ_t, and δ are respectively the normal rotation, twisting rotation, and out-of-plane displacement, along a yield-line or at the edge of a slab). Also, **N** is a suitable plastic flow matrix and $\mathbf{f}_L$ is a vector that prescribes the effect of live loads 'above' each yield-line.

The optimization variables are the yield-line displacements in **d** and plastic multipliers in **p**. Since all terms are linear, the optimization formulation (1) can be solved using linear programming (LP). The entire optimization problem can be assembled using locally derived formulae for each yield-line, which are introduced in the following section.

2.2. Terms for a single yield-line

For a yield-line i that connects two nodes A(x_A, y_A) and B(x_B, y_B), and inclined at an angle ϕ to x axis, as shown in Fig. 3, let $x_l = x_B - x_A$ and $y_l = y_B - y_A$. (Note that in the interests of conciseness, the subscript i has been omitted, i.e. x_l is used rather than x_{li}; this is repeated for all coefficients defined in this section). The length of this yield-line is calculated using $l = \sqrt{x_l^2 + y_l^2}$, so $\cos\phi = x_l/l$. Now assume that the displacement variables in **d** for this yield-line are of the form $[\theta_n, \theta_t, \delta]^T$. The contribution to the nodal compatibility constraint (1b) for this yield line is given by: