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A peridynamic theory for linear elastic shells

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ABSTRACT

A state-based peridynamic formulation for linear elastic shells is presented. The emphasis is on introducing, possibly for the first time, a general surface based peridynamic model to represent the deformation characteristics of structures that have one geometric dimension much smaller than the other two. A new notion of curved bonds is exploited to cater for force transfer between the peridynamic particles describing the shell. Starting with the three dimensional force and deformation states, appropriate surface based force, moment and several deformation states are arrived at. Upon application on the curved bonds, such states yield the necessary force and deformation vectors governing the motion of the shell. By incorporating a shear correction factor, the formulation also accommodates analysis of shells that have higher thickness. In order to attain this, a consistent second order approximation to the complementary energy density is considered and incorporated in peridynamics via constitutive correspondence. Unlike the uncoupled constitution for thin shells, a consequence of a first order approximation, constitutive relations for thick shells are fully coupled in that surface wryness influences the in-plane stress resultants and surface strain the moments. Our proposal on the peridynamic shell theory is numerically assessed against simulations on static deformation of spherical and cylindrical shells, that of flat plates and quasi-static fracture propagation in a cylindrical shell.

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1. Introduction

Great leaps in computing power over the last few decades have enabled researchers to undertake numerical simulations of myriad complex, multi-scale and multi-physics problems of interest in continuum solid mechanics that were erstwhile unsolvable. Numerical techniques such as the finite element method (FEM) as well as mesh-free discretization schemes, e.g., smoothed particle hydrodynamics (SPH), reproducing kernel particle method (RKPM), moving least square Petrov–Galerkin method (MLPG) and so on have provided the necessary formalism in obtaining finite dimensional approximations to solutions of such initial boundary value problems (IBVP) that describe, often in terms of partial differential equations (PDEs), the laws of motion. Although PDE-centric formulations have been successful in modelling phenomena ranging from elastic response of solids to more complicated problems involving, for instance, propagating discontinuities in dynamic crack propagation, void growth and contact mechanics, its formal mathematical structure does not provide an ideal setup for applications to scenarios that must deal with evolution of discontinuities, e.g. crack nucleation and growth and other such problems in continuum damage mechanics. The partial derivatives in PDEs are not defined in the classical sense on the lines/surfaces of discontinuities and no valid diffeomorphism is at hand to relate the deformed and reference configurations. Computational methods for solving such problems using the PDE-based theory either require a redefinition of the object manifold so that discontinuities lie on the boundary or some special treatment to define spatial derivatives of field variables on a cracked surface (see Bittencourt et al., 1996, Belytschko and Black, 1999 and Areias and Belytschko, 2005).

Recently, Silling (2000) introduced a reformulation of continuum theory, namely, the peridynamics (PD), which can, by design, treat discontinuities. The primary feature that enables PD to deal with spontaneous emergence and propagation of discontinuity in solids, is the representation of equations of motion in integro-differential form, and not by PDEs. The integro-differential format of the governing equations relaxes, to a significant extent, the smoothness requirement of the deformation field, thereby allowing for discontinuities as long as the spatial integrals remain Riemann-integrable. PD equations of motion are based on a model of internal forces that the material points exert on each other over finite distances. Such finite distance interactions lend a non-local character to the formulation and allow for length scale effects arising from the action at a distance. This inherent non-local feature in PD theory is useful in modelling a broad class of non-classical phenomena. An initial formulation, the bond-based PD, considered

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internal forces as a network of interacting pairs like springs, that is, it described spring-like interactions via pair potentials. The interaction of a material particle with its surroundings is restricted to a finite neighbourhood and is denoted as the horizon of the particle. Such pair-wise interaction however led to an over-simplification of the model and in particular resulted in an effective Poisson's ratio of 1/4 in case of linear isotropic elastic materials. This limitation has been overcome through a generalization of the PD model, the state-based PD (Silling et al., 2007). According to the state-based PD philosophy, the bond force between two interacting particles is no longer governed by a central potential independent of the behaviour of other bonds; instead it is determined by the collective deformations of bonds within the horizon of a material particle. This version of the PD theory eliminates the restriction on Poisson's ratio and is applicable over the entire permissible range. Even though the PD has many attractive features, the scarcity of strictly PD-based material constitutive models tends to limit its applicability. A remedy to this is however proposed using a constitutive correspondence framework (Silling et al., 2007), enabling the use of classical material models in a PD formulation.

These features of the PD have attracted interest in solving solid mechanics problems especially those involving material damage. Most such attempts deal with the full-blown 3D model of continua, whilst a few rest consider the in-plane response within plane stress or plane strain type material modelling. Even though examples of structures resisting transverse deformation with one dimension (e.g. the thickness) significantly smaller than the other two are aplenty (e.g. aircraft fuselage, ship hull, pressure vessel, roofs of civil structures, turbine blades and so on), very few attempts in the PD literature are available that exploit the possibility of efficiently modelling such 3D bodies in terms of locally 2D equations of motion. To cite some instances of dimensionally reduced PD models, we refer to Silling et al. (2003) for 1D bar formulation, Chowdhury et al. (2015) and O'Grady and Foster (2014a)for 1D beams, Silling and Bobaru, (2005) for 2D membranes, Taylor and Steigmann (2013), O'Grady and Foster (2014a) and Diyaroglu et al. (2015) for plates and flat shells. Such studies are useful to relieve the extensive computational overhead generally associated with the discretization of a 3D model. For instance, if the so-called thickness dimension is small, use of the 3D model would typically demand a rather fine discretization in the through-thickness direction *en route* to an accurate representation of the resistance to bending. This may need prohibitively expensive computational effort. The 2D formulations address this issue by an analytical accounting of the stress and deformation fields along the thickness direction and thus avoid through-thickness discretization. Studies by Taylor and Steigmann (2013), O'Grady and Foster (2014b) and Diyaroglu et al. (2015) reflect a few such efforts, with the first reducing a 3D bond-based PD formulation to 2D in order to model bending characteristic of plates, the second deriving a state-based PD model for plates and flat shells and the third offering another bond based formulation for plates. While the bond based plate formulation of Taylor and Steigmann suffers from the usual limitation of Poisson's ratio being reduced to 1/4, O'Grady and Foster's state-based model is applicable for the entire permissible range of Poisson's ratio. Even though the three models noted above can analyse transverse deformation of flat structures, deformation of a 'thin' 3D body that may be described with reference to a curved base surface cannot be analysed by them. Owing to the curvature of the surface, analysis of such structures is more complicated than that of flat structures, as transverse bending effects generally get coupled with stretching. Moreover a flat plate model may always be recovered as a limiting case of a curved shell model. To the best of the authors' knowledge, there exists no generic surface-based PD formulation to deal with such problems. In contrast, the literature on classical continuum mechanics is rich with an abundance of shell models. In classical shell modelling, kinematics and kinetics of the 3D body are described through tensor quantities defined over a base surface, typically the mid surface. The governing equations of motion and constitutive relations for a shell, in terms of these surface based tensor quantities, may be arrived at via through-thickness integration of the 3D equations of motion and constitutions, respectively. Shell equations so obtained include only two independent surface coordinates vis-a-vis three independent ones in the 3D model. Strain measures referred to the base surface may be obtained from the power balance equation as quantities conjugate to shell stress measures. Assumed variation of the displacement field along the thickness direction is made use of in identifying the appropriate shell strains (Reissner, 1941, Naghdi, 1973, Reddy, 2007).

Depending on their thickness, shells are categorised as either thin or thick. While shear deformation is negligible for thin shells, it does play an important role in describing the deformation kinematics of thick shells. Even when a shell is 'thick', thickness is generally not large enough to warrant a full-fledged 3D continuum model. Classical analyses of thick shells typically follow a Reissner type hypothesis that allows for shear deformation. In order to derive shell constitutive relations reflecting an acceptably accurate distribution of transverse shear stress, the 3D complementary energy density with a suitable splitting of stress components is made use of. Following series expansions of the 3D stress components in the thickness direction in terms of the surface force, transverse force and moments, through-thickness integration and a subsequent Fréchet derivative of the effective complementary energy density leads to the dimensionally reduced constitutions. Transverse forces are expressed in a manner that accounts for their possible variations across the thickness. The detailed information on the through-thickness variation is however lost and only averaged quantities are retained. The conflict arising out of the inherently incongruent nature of the shear force (which is a stress resultant) and the prescribed traction boundary conditions on the shell surface is often avoided by advocating a shear correction factor in the classical theory of thick shells. Exclusion of this factor shows up in the unphysical feature of the response being stiffer. Another important aspect in this theory is the presence of terms that couple surface force and moment measures in the complementary energy density. The coupling terms are significant when the curvature of the base surface is substantial; see Pietraszkiewicz et al. (2006, 2014).

In the present work, a surface based PD formulation, possibly the first of its kind and applicable for general curved shells, is set forth. A non-ordinary state-based approach is adopted. The 3D state-based PD equations are reduced to their surface representations by defining new force and moment state fields obtainable from 3D force states upon appropriate integration over the thickness direction. The proposed set of equations that describes the motion of shell is shown to satisfy the global requirements of linear and angular momenta balances for the 3D body. New deformation state fields referred to the base surface are identified from the energy balance equation. These states appear as conjugate quantities to the force and moment states in internal energy expression. A new notion of curved bonds replacing straight bonds in the standard PD theory is introduced. The curved bonds facilitate transfer of force and moment between PD particles. Also the 'size' of the horizon of a particle, which is of curved surface geometry, is decided by fixing a maximum length of curved bonds within it. In order to model the shear deformation effect in thick shells, a splitting of the 3D transverse force and deformation vector states is carried out in the PD power balance equation. Integrating along the thickness, deformation states are then identified as conjugate to the corresponding force states. The force state in the transverse direction and the corresponding deformation state are separately

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