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A unified model for the contact behaviour between equal and dissimilar elastic–plastic spherical bodies

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ABSTRACT

A unified method for calculating the contact force and the contact area between two dissimilar elastic-plastic spheres is presented with the aim of simulating granular materials using particle methods. Explicit equations are presented for the case when the plastic behaviour of the spheres is described with three material parameters. This makes the analysis applicable for a wide range of materials. The model is partly based on dimensionless quantities emerging from the Brinell hardness test. Large deformation of the contact is accounted for in the analysis, which allows for accurate contact relations up to indentation depths relevant for powder compaction. The presented model shows excellent agreement with finite element simulations of two spheres in contact and the results found in literature. An implementation of the contact model in Python is provided together with the online version of this paper.

1. Introduction

The mechanical problem of two deformable spheres in contact has gained increasing interest during the recent years due to, among other things, its importance in simulations of granular materials and flattening of surface asperities. However, the first study of the problem and, arguably, the onset of contact mechanics as a specialised research subject, is the prominent paper by Hertz (1882) about contact between elastic spherical bodies, which serves as a landmark in the theory of elasticity.

Following the work by Hertz (1882), elastic contact mechanics has evolved considerably and, as an example, the works by Sneddon (1965) and Spence (1975), concerning non-spherical contact and frictional effects, should be mentioned. Mathematical techniques based on complex variables, integral transforms and Green functions have been essential for this development. The progress related to elastic contact mechanics has been summarized by for example Gladwell (1980) and Hills et al. (1993).

Contact between inelastic bodies is a much more involved problem and much of the development has been driven by the aim of determining material properties from indentation testing, with important contributions from Tabor (1951) and Johnson (1970, 1985). These have received increasing attention and appreciation, due to the development of new experimental devices like the nanoindenter, (Pethicai et al., 1983), enabling an experimentalist to determine the material properties from extremely small samples of the material. With the advent of modern computers and advanced numerical methods (in particular the finite element method (FEM)), indentation tests have been extensively analysed. The early works by Hardy et al. (1971), Lee et al. (1972), Bhattacharya and Nix (1988a, 1988b), Laursen and Simo (1992), Giannakopoulos et al. (1994) and Larsson et al. (1996), deserves a mention in this context. The combination of experimental and numerical studies of indentation problems has resulted in considerable progress and increased understanding.

Regarding numerical studies of contact problems, one issue, from a finite element perspective, is the moving contact boundary during the indentation process. This requires a dense FE mesh over the whole contact area, and in particular so close to the contact boundary, and thus comparatively large computational times. Such a problem can be avoided if the possible self-similarity of the contact problem can be exploited. If, the indenter and the stress-strain relationship in an indentation problem can be described by power law functions, it is possible to reduce the problem to a rigid flat punch indenting a deformable half-space, cf. e.g. Borodich (1993) for nonlinear elasticity. The self-similarity of the problem has been utilized in studies of Brinell indentation assuming deformation plasticity (Hill et al., 1989), power-law creep (Storåkers and Larsson, 1994) and plastic flow theory (Biwa and Storåkers, 1995). It should be mentioned though that in the latter studies, self-similarity can only be assumed if elastic deformations are neglected.

Following the early efforts discussed above, numerous studies of indentation and other similar contact problems have been presented based on numerical methods and in particular FEM. Important contributions to the understanding of spherical contact was presented

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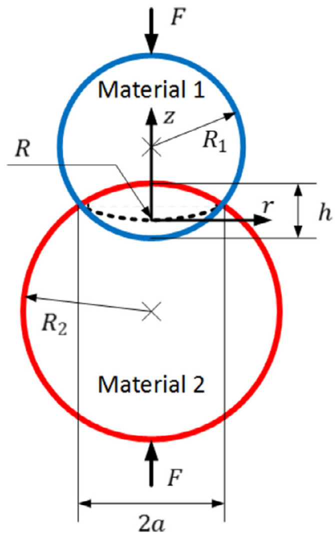

Fig. 1. Sketch of the studied contact problem.

by Mesarovic and Fleck (1999, 2000) where it was shown that self-similarity solutions relying on the assumption of purely plastic deformations have a rather limited region of validity. This is due to the influence from elasticity and large deformations. Based on the results by Mesarovic and Fleck (1999, 2000), Olsson and Larsson (2013c) presented a comprehensive study where global contact quantities, in spherical contact, were correlated for the case of plastic flow theory while also accounting in full for elastic effects. Olsson and Larsson (2013c) presented a complete set of closed-form solutions for the global contact quantities with the only constitutive restrictions being that power law strain-hardening was assumed. It should be noted though that large deformation effects, at high values of the ratio of indentation depth to characteristic length of the indenter, were not considered.

The relation between contact force and indentation depth (in a general case interpreted as the mutual approach of the two bodies in contact) is a very important quantity given by a contact mechanics analysis. It is relevant not only for material characterization by nanoindentation but also for other contact problems. For example, the accuracy of micromechanical analysis of powder compaction is almost completely determined by the accuracy of the force–displacement relation between two contacting particles, cf. e. g. Storåkers et al. (1999), Martin (2004) and Olsson and Larsson (2013a). There are numerous attempts in literature to determine such relations, (Brake, 2015; Kogut and Etsion, 2002; VuQuoc et al., 2001), with varying degrees of accuracy and generality. Recently, Olsson and Larsson (2013b) presented an analysis of spherical contact between elastic–plastic bodies, which included a rigorous treatment of both elastic and plastic effects ranging from Hertzian contact to fully plastic deformation. In this study, high accuracy solutions were obtained in the situation of low and intermediate values of the indentation depth. It was observed, that the description of global contact properties at elastic and plastic deformations of equal magnitude had to be improved in order to increase the accuracy of the solution.

Accordingly, it is the aim of the present study of the spherical contact problem, sketched in Fig. 1, to improve upon this situation. The analysis by Olsson and Larsson (2013b) will serve as the theoretical background while results of higher accuracy are achieved by using global contact relations determined from relevant finite element simulations. Furthermore, large deformation effects are included in the analysis in order to extend the region of validity into high values on the ratio of indentation depth to characteristic length of the spherical bodies. From a practical viewpoint, this leads to, for example, an extension of the region of validity of micromechanical analysis of powder compaction into high density regimes. The analysis is restricted to classical elastic–plastic materials but the effects from elasticity and plasticity are accounted for completely. The outcome of the investigation is compared with full-field finite element simulations of the problem for a wide variety of geometrical and material combinations.

2. Theoretical background

The two key parameters in the present analysis are the Brinell hardness of the materials involved in the contact problem and a parameter relating the contact area under a spherical indenter to the indentation depth. The analysis will be based on the evolution of these parameters during the indentation process. In the classical work by Tabor (1951), it was concluded that the hardness H, for different materials at fully plastic indentation could be calculated from

$$H = \frac{F}{\pi a^2} = \bar{H}\sigma_{rep} \tag{1}$$

where the *normalized* hardness, $\bar{H}$, takes on the value 2.8 and the representative stress, σ_{rep}, should be chosen to be the yield stress of the material at a representative value of the effective (accumulated) plastic strain ε_{rep} as

$$\varepsilon_{rep} = 0.2\frac{a}{R_{ind}} \tag{2}$$

where R_{ind} is the radius of the indenter. Although, using the self-similarity model, (Biwa and Storåkers, 1995), the results by Tabor (1951) was confirmed theoretically, it was suggested that an even better correlation regarding the hardness could be achieved if

$$H = 3.07\sigma(\varepsilon_{rep} = 0.32a/R_{ind}). \tag{3}$$

For harder materials, the assumption of fully plastic indentation does not apply and the value $\bar{H} = 2.8$ will predict a too high hardness. Based on the assumption of a hydrostatic core under the indenter, Johnson (1970) was able to correlate the outcome of an indentation test to a single master curve for general elastoplastic materials. It was found that the state of the spherical contact problem falls under three different regimes depending on a dimensionless parameter Λ, which is defined as

$$\Lambda = \frac{E}{(1-\nu^2)\sigma_{rep}}\frac{a}{R_{ind}} \tag{4}$$

where E and ν are the elastic modulus and Poisson's ratio respectively, a is the projected radius of the contact surface and R is the radius of the indenter. This parameter can be interpreted as a quotient between the deformation (a/R) and the elastic capacity of the material. A corresponding expression for Λ exist for sharp contact problems.

The spherical indentation problem can be characterized using two dimensionless parameters, the previously normalized hardness $\bar{H}$, and the contact area parameter c^2, defined as

$$c^2 = \frac{a^2}{2hR_{ind}} \tag{5}$$

The behaviour of these two parameters as function of Λ is sketched in Fig. 2 together with markings of the three different indentation regimes.

In the first regime (Level I), valid for $\Lambda < 3$, i.e. at elastic contact, plastic deformations can be neglected and $\bar{H}$ and c^2 are given by Hertzian contact theory (Hertz, 1882). In the second regime (Level II), both elastic and plastic effects need to be accounted for in order to describe the contact with approximately linear behaviour of $\bar{H}$ and c^2 as a function of $\ln\Lambda$ as indicated in Fig. 2. The last regime concerns fully plastic contact (Level III) where elastic effects can be neglected. This is the region where the self-similarity solution applies,